1,2 Simple Explanation of Einstein's Relativity Theory

Adrian Bjornson (May 2009)

Measuring the Speed of Light

Einstein's Relativity theory evolved to explain an enigma associated with measuring the speed of light. To introduce this issue, let us first consider measuring the speed of sound. Light travels at a speed of 300,000 kilometers per second (km/sec). This is about one million times faster than the speed of sound, which is approximately 340 meters per second (m/sec).

Sound is a vibration of the air. It travels as a wave through the air, somewhat like the way a wave travels across the surface of a still pond, when you throw a rock into it. If the air is stationary (no wind), an instrument on the ground will measure the speed of sound as 340 m/sec. If the wind speed is 20 m/sec (72 km/hour or 45 mph) the instrument will measure a speed of sound of (340 + 20) m/sec, or 360 m/sec if the sound is travelling in the direction of the wind, and it will measure a sound speed of (340 – 20) or 320 m/sec, if the sound is travelling in opposition to the wind. Similarly, if the instrument is placed on a vehicle travelling at 20 m/sec, and there is no wind, the instrument will measure a sound speed of (340 + 20) or 360 m/sec if the vehicle is travelling opposite to the direction of the sound, and it will measure a sound speed of (340 – 20) or 320 m/sec, if the vehicle is travelling in the direction of the sound wave.

It was originally assumed that light travels as a wave by vibrating a mysterious medium, called the aether, just as sound travels by vibrating the air. In 1873 Maxwell presented his famous electro-magnetic theory, which formed the mathematical basis for designing radio, radar, and television systems, and all of our modern electronic devices. His theory showed that light is a packet of oscillating electrical and magnetic fields, which does not require a medium to allow propagation. But without a medium, what does one mean by the speed of light? Is light speed measured relative to the transmitter, or relative to the receiver, or maybe relative to an aether medium? Apparently because of this question, Maxwell included the aether concept in his theory, even though an aether was not needed in the propagation of light, a radio wave, or any other electromagnetic radiation.

In 1887, the famous Michelson-Morley experiment was performed to measure the velocity of the aether "wind". The speed of light from a star close to the plane of the earth's orbit was measured. Since the earth travels at 30 km/sec in its orbit around the sun, at one point of the orbit the earth is travelling toward the star at 30 km/sec, and six months later it is travelling away from the star at 30 km/sec. If the aether is stationary relative to the sun, the speed of light should presumably be (300,000 + 30) or 300,030 km/sec when the earth is moving toward the star, and (300,000 – 30) or 299,970 km/sec when the earth is moving away from the star. However, there was absolutely no change in the speed of light measured at different points in the earth's orbit.

Additional measurements confirmed the fact that the speed of light was always exactly the same, regardless of whether it is measured relative to the transmitter or the receiver, even though there was an appreciable velocity between the transmitter and the receiver. Scientists were struggling to explain this enigma when Einstein published his Theory of Relativity in 1905.

The Einstein Theory of Relativity

Einstein explained that when there is a relative velocity (V) between two observers, the clock of the other observer appears to run more slowly by a contraction factor K given as follows, where c is the speed of light:

$$K = \sqrt{[1 - (V/c)^2]} \quad \text{(contraction factor)}$$

To each observer, the clock of the other observer appears to run at a rate equal to (K) times the rate of one's own clock. Also, a measuring rod moving with the other observer appears to be compressed in length by this factor (K), when pointed in the direction of the relative velocity. Finally, two clocks of the other observer, which are synchronized to the other observer, appear to be out of synch by the amount [(V/c)(D/c)], measured relative to the other observer's clock, where D is distance between the clocks in the direction of the relative velocity. When these three effects are applied, it can be shown that the two observers always measure the same value for the speed of light, regardless of the velocity (V) between them.

The fundamental result of the Einstein relativity theory is that time is not absolute; it is relative. Based on the relativity of time measurement, Einstein derived some revolutionary concepts, including his famous equation: ($E = Mc^2$). This equation showed that mass and energy are equivalent, and that mass (M) can be converted into energy (E) according to this formula. This formula explained the source of the enormous energy radiated by the sun. The sun converts hydrogen into helium, which reduces the mass by 0.71 percent, and this loss of mass is converted into 177,500 kilowatt-hours of energy per gram of hydrogen. Later, this formula was the theoretical foundation for developing the atomic nuclear bomb during World War II.

Einstein concluded that the speed of light is constant, that electromagnetic radiation cannot travel faster than light, and that a body with mass can never reach the speed of light. This principle is readily demonstrated by accelerating an electron in an electric field. Until recently, all television displays used cathode-ray tubes, which we call picture tubes. Inside the glass envelope of a cathode ray tube is a vacuum, which allows electrons to flow freely. The heated cathode boils off electrons, which are attracted to a wire grid that has a high positive voltage relative to the cathode. The electric field between the cathode and the grid accelerates the electrons to an appreciable velocity. A varying magnetic field deflects the electrons to different points on the face of the picture tube to produce the television picture.

A similar principle can be applied to accelerate electrons to velocities approaching the speed of light. Electrons in a vacuum chamber are boiled off from a cathode and accelerated in an electric field. Then they are deflected magnetically around a circle and fed through the electric field again. Each time an electron passes through the electric field, its energy is increased by the same amount. Initially the increase of energy results in a corresponding increase in the electron velocity. However, when the electron velocity gets close to the speed of light, the electron mass begins to increase appreciably, and the velocity increases more slowly. The energy derived from the electric field is converted into electron mass according to the Einstein formula ($E = Mc^2$). The electron becomes heavier and heavier as its velocity gets closer and closer to the speed of light. The electron velocity never reaches the speed of light, because this would require infinite mass in the electron.

We normally consider spatial and time measurements to be independent. However, Einstein showed that this assumption is only approximately valid, and does not hold with reasonable accuracy when high velocities are involved. When observers travelling at different velocities observe two events, a time interval between the events measured by one observer can appear to be a spatial interval to the other observer. Spatial and time measurements are inter-related, and so spatial and time measurements must be combined together to form a space-time specification that is unique. With space having three dimensions and time having one dimension, space-time has four dimensions. To achieve a unique specification, reality must be specified in four dimensions.

I remember as a child thinking of the mysterious fourth dimension, and wondering which direction in space could possibly represent a fourth dimension. I now realize that I was asking the wrong question. To any observer, spatial and time dimensions are entirely separate concepts. Each observer experiences three spatial dimensions and one time dimension. However, space and time measurements must be combined to obtain a rigorous four-dimensional specification of reality, so that the measurements of observers moving at different velocities can be compared.

Effects of Acceleration and Gravity

The relativity theory presented by Einstein in 1905 was based on the postulate that the speed of light is constant. This clearly holds when observers are travelling at constant velocity, but what happens when the velocity changes, when acceleration occurs? Einstein performed approximate calculations, which proved that the speed of light is not constant under conditions of acceleration. He also concluded that acceleration and gravity are basically the same, and that any effect produced by acceleration must also occur in an equivalent gravitational field. He realized that he had to incorporate acceleration and gravity into his relativity theory in order to achieve a rigorous theory. He developed his basic relativity theory in a few months, but it would take eleven hard years of research to generalize his relativity principle and thereby include the effects of acceleration and gravity.

Let us examine an approximate calculation that Einstein performed to prove that acceleration and gravity cause the speed of light to change. Figure A-1 shows two elevators: elevator (b) is fixed on earth; elevator (a) is in space and is pulled upward with an acceleration equal to the acceleration of gravity (g) on earth. Today we would assume that elevator (a) is forced upward by a rocket with a one-G acceleration. Because the acceleration is one-G, a person in elevator (a) would feel an acceleration force on his body equal to the weight force he would feel when in the fixed elevator (b) on earth. Einstein proposed his ***Principle of Equivalence***, which postulates that physical experiments within the two elevators would produce exactly the same results, and so the effects of acceleration and gravity are identical.

Suppose a light pulse is emitted from the floor of elevator (a) at point (1) and is received at the ceiling at point (2). During the propagation time of the light, the velocity of the elevator increases because the elevator is accelerating. Hence the upward velocity of the receiver at point

(2) is greater than that of the emitter at point (1) as far as the light pulse is concerned. Point (2) appears to be moving away from point (1), and so the light spectrum is shifted toward the red as it moves from point (1) to point (2). (Astronomers call this a “redshift”.) Since the effects in elevators (a) and (b) are the same, the gravitational field in elevator (b) must also produce a spectral redshift as light propagates from the floor to the ceiling.

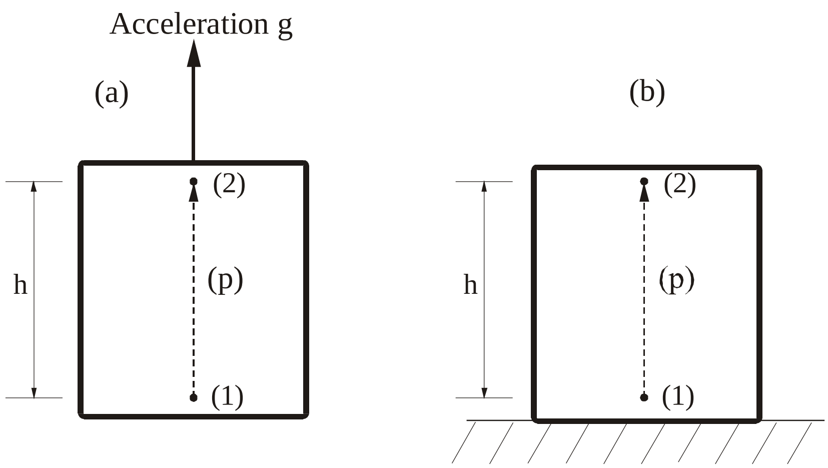

Figure A-1: Elevator (a) is accelerating in free space; elevator (b) is fixed on earth. Light travels from point (1) to point (2).

A light wave can be used to time a clock, which gives one tick for every period of the light wave. When a light wave is shifted toward the red, its period increases, and so a clock synchronized to the light wave must tick more slowly. Hence a clock at the ceiling of the elevator runs more slowly than an identical clock on the floor. In other words, raising a clock in a gravitational field causes the clock to tick more slowly. The fractional increase of the clock period can be shown to be approximately equal to (gh/c^2), where g is the acceleration of the gravitational field, h is the height of the ceiling above the floor, and c is the speed of light.

Einstein performed another more complicated approximate analysis using his two elevators, which showed that a gravitational field causes a spatial dimension to contract. The fractional decrease of a spatial dimension between the floor and the ceiling of the elevator is also approximately equal to (gh/c^2).

The speed of light is equal to (D/T), where (D) is the distance traveled by the light and (T) is the time for the light to travel. If the period (T) of a clock increases, the speed of light is reduced; and if a spatial dimension (D) decreases, the speed of light is also reduced. Hence both effects contribute to a reduction of the speed of light. The relative reduction of the speed of light is approximately equal to the sum of these effects, which is $2(gh/c^2)$.

Thus, from his approximate analyses Einstein found that acceleration or gravity causes the speed of light to decrease by a fractional amount approximately equal to $2(gh/c^2)$. This proved that the speed of light is not constant under conditions of acceleration or gravity, and so Einstein knew that he must generalize his relativity theory. After Einstein published his generalized theory in 1916, which he called General Relativity, his basic relativity theory published in 1905 was called Special Relativity.

Principles for Generalizing Relativity

To obtain a rigorous general theory of relativity, Einstein needed a radically different mathematical foundation. He found this in an elaborate mathematical theory of curved space published in 1901, which had been developed by the Italian mathematician Gregorio Ricci (1853-1925), and was based on a mathematical principle presented in 1852 by the German mathematician Bernhard Riemann (1826-1866). This mathematical theory is commonly called “Riemannian geometry”, and is usually attributed only to Riemann. However, the Ricci theory is actually a sophisticated mathematical theory that extends differential calculus to curved space. Ricci called it “The Absolute Differential Calculus”. Although this mathematical theory is based on Riemann’s principle, it is a highly extensive and rigorous expansion of that principle. For this reason, it should be properly called the ***Ricci-Riemann calculus of curved space***.

Gregorio Ricci was assisted in developing his theory by his pupil, Tullio Levi-Civita (1873-1941). In 1923, Levi-Civita published in Italian an updated version of this mathematical theory. An English translation is available as a Dover reprint. [5] ***I personally feel that it is a disgrace that the enormous contributions of Ricci and Levi-Civita to General Relativity theory have been largely ignored by the scientific community.***

Einstein incorporated gravity (along with acceleration) into his theory by expressing it as a curvature of space. The Ricci-Riemann calculus of curved space could handle multiple dimensions, and so Einstein applied this mathematical theory in four dimensions to represent the four-dimensional space-time requirements of relativity. Einstein used the effect of gravity to establish the curvature of this four-dimensional Ricci-Riemann calculus of curved space.

We can illustrate a simple curved space by considering motion across the spherical surface of the earth. In flat Euclidean space, a straight line is the shortest distance between two points. In curved space, the shortest distance between two points is called a ***geodesic***. On the curved surface of the earth, the shortest distance between two points is a great-circle route, which represents a geodesic. A great circle route is constructed by passing a plane through the center of the earth and the two points. The geodesic great-circle route is the intersection of this plane with the spherical surface of the earth.

For example, Boston, Massachusetts and Rome, Italy are at nearly the same latitude. One might think that an airplane from Boston to Rome would fly directly east, but that is the long way. Along the shorter great-circle geodesic route, the airplane starts flying nearly north-east from Boston, and ends the flight by flying nearly south-east to reach Rome.

Let us consider the effect of gravity on the earth as it rotates around the sun. According to Newton’s law of motion, a body moves at constant velocity along a straight line, unless a force is applied to it. In terms of Newton’s theory, the earth moves in a curved orbit around the sun because the gravitational force from the sun keeps the earth from flying off into space along a straight line. ***But, there is no gravitational force in the Einstein gravitational theory.*** In the

Einstein theory, the earth travels along a geodesic path in curved space, which is equivalent to a straight line in flat Euclidean space, and is the shortest distance between two points. The gravitational field of the sun curves the space around it, and the earth moves within this curved space along a geodesic path. The Ricci-Riemann calculus of curved space has a general equation, called the ***geodesic equation***, which allow one to calculate the geodesic path that is followed by the earth as it orbits around the sun. The geodesic equation is given by this website in the *Addendum* document *5,4 Application of Geodesic Equation to the Yilmaz Theory*.

The curvature of space produced by gravity is a four-dimensional space-time effect. Consequently, the geodesic path followed by a body in the gravitational field of the sun depends on time as well as on spatial location. Consider a planet at a particular location in the gravitational field of the sun. The geodesic path that the planet follows depends not only on the position of the planet but also on the rate-of-change of position relative to time, which is the velocity of the planet.

Mathematical Calculations of Relativity

Metric Tensor. The Ricci-Riemann calculus of curved space is expressed in terms of tensors. A simple physical description of a tensor is shown in Appendix A. In the four-dimensional Einstein application, the tensors are 4x4 arrays, and so have 16 separate elements. The fundamental tensor of the Ricci-Riemann calculus is the ***metric tensor***, which specifies the measurement properties of curved space. The metric tensor is denoted g_{ab}, where the indices a, b can be 0, 1, 2, or 3. The index 0 represents the time dimension, and 1, 2, 3 represent the three spatial dimensions, which are often denoted x, y, and z. The elements of this tensor are:

$$\begin{vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{vmatrix}$$

The metric tensor is symmetric, meaning that $g_{12} = g_{21}$, $g_{23} = g_{32}$, etc. Because of this symmetry, six of the elements are redundant, and the tensor has only 10 independent elements. The elements along the diagonal of the array (g_{00}, g_{11}, g_{22}, g_{33}) are called the ***diagonal*** elements, and the others are called ***non-diagonal***. If all of the non-diagonal elements are zero, the tensor is called ***diagonal***.

If the metric tensor is not diagonal, the equations of General Relativity are extremely complicated. They literally have millions of terms, and so cannot be solved analytically. Consequently, Einstein could apply his theory only to very simple physical models that yielded diagonal metric tensors. Einstein died in 1955. It was not until the mid 1960's, a decade after Einstein's death, when powerful computers became readily available, that General Relativity could be applied to more complicated physical models that yielded non-diagonal metric tensors. Hence, if we restrict ourselves to physical models that Einstein could study, there are only four elements of the metric tensor that we need consider: the diagonal elements g_{00}, g_{11}, g_{22}, and g_{33}.

Often the three spatial elements, g_{11}, g_{22}, g_{33}, are equal, and we need to consider only two separate elements, the time element g_{00} and the spatial element g_{11}.

From the values of the metric tensor, one can directly calculate many relativistic effects. The metric equation, which is based on the metric tensor, is explained by this website in *1,7 Metric Equation*. The metric equation yields the following formulas that show how a gravitational field changes the speed of light, a spatial dimension, and a clock period:

$$(c_{ap}/c) = \sqrt{[-g_{00}/g_{11}]} \qquad \text{(speed of light)} \qquad (3)$$

$$(\Delta x_{ap}/\Delta x) = 1/\sqrt{[-g_{11}]} \qquad \text{(spatial dimension)} \qquad (4)$$

$$(\Delta T_{ap}/\Delta T) = 1/\sqrt{[g_{00}]} \qquad \text{(clock period)} \qquad (5)$$

The Greek symbol (Δ) is called "delta" and is used to denote a difference. Symbols c_{ap}, Δx_{ap}, and ΔT_{ap} denote the ***apparent*** values observed here on earth for the speed of light, a spatial dimension, and a clock period that occur at a distant location. Symbols c, Δx, and ΔT are the corresponding ***true*** values that would be measured by an observer placed at the distant location. (Einstein used the term ***coordinate value*** instead of ***apparent value***, and the term ***proper value*** instead of ***true value***.) These equations assume that the metric tensor is diagonal and that the three spatial elements g_{11}, g_{22}, g_{33} are equal. The equations show that, when the elements of the metric tensor are known, one can readily calculate some important relativistic effects of gravity.

The tensor g_{ab} is called the ***covariant*** form of the metric tensor. There is another form denoted g^{ab} that is called the ***contravariant*** form of the metric tensor. The product of the covariant and contravariant metric tensors is a unit matrix. If the covariant metric tensor is diagonal, the contravariant form is also diagonal, and its diagonal elements are the reciprocals of the corresponding covariant elements (*i.e.*, $g^{00} = 1/g_{00}$, $g^{11} = 1/g_{11}$, etc.)

Forms of Other Tensors. Other tensors of the Einstein theory have three forms, called ***covariant***, ***contravariant***, and ***mixed***. For a general covariant tensor denoted D_{ab}, the contravariant form is denoted D^{ab}, and the mixed form is denoted $D_a{}^b$. The Ricci-Riemann calculus of curved space has general formulas (using elements of the metric tensor) for computing one of these forms from another. (These formulas are given in *Believe* [1], Appendix A.) The metric tensor also has a mixed form denoted $g_a{}^b$, but is mathematically trivial.

The Ricci Curvature Tensor $R_a{}^b$. The Ricci tensor describes the curvature of space. If there is no gravitational field or acceleration in a region of space, that region has no curvature, and all elements of the Ricci tensor in that region are zero. The Ricci tensor is usually applied in its mixed form $R_a{}^b$. The Ricci-Riemann calculus of curved space provides general equations for calculating the covariant Ricci tensor R_{ab} from the metric tensor, which are given in *Believe* [1], Appendix A. The covariant Ricci tensor R_{ab} is then converted to its mixed form $R_a{}^b$ by means of formulas that use the metric tensor.

The Einstein Tensor $G_a{}^b$. In General Relativity, the curvature of space is usually expressed in terms of the Einstein tensor $G_a{}^b$, which is a modification of the Ricci tensor. The Einstein tensor is defined by ($G_a{}^b = R_a{}^b$) for the non-diagonal elements of the tensor, and by ($G_a{}^b = R_a{}^b - ½ R$) for diagonal elements, where R is the sum of the diagonal elements of the mixed Ricci tensor $R_a{}^b$.

The reason for converting from the Ricci tensor to the Einstein tensor is that the ***covariant derivative*** of the Einstein tensor is zero, whereas that of the Ricci tensor is not. The covariant derivative is explained in *Believe* [1], Appendix J. The covariant derivative of a vector or tensor describes the true change of a vector or tensor in curved space. For example, consider a vertically pointing vector located at a particular point on the earth. Assume that a second vector is kept parallel to the first vector, while being moved to a different location. Because the surface of the earth is curved, the second vector will no longer be vertical. Its coordinates relative to the curved surface of the earth have changed, but the absolute direction of the vector is the same. When the covariant derivative of a vector is zero, the absolute direction of the vector does not change when it is moved to a different location.

When the metric tensor is diagonal, the calculations discussed above can be greatly simplified by applying general formulas that were derived by Prof. Herbert Dingle. These are presented in Section B.5 of the *Addendum* document *5,B Einstein Tensor and Christoffel Symbols for Diagonal Metric Tensor*. The elements of the mixed Einstein tensor $G_a{}^b$ are expressed in terms of the elements of the covariant metric tensor g_{ab}. A look at these Dingle formulas shows that the equations of General Relativity are extremely complicated, even when the metric tensor is diagonal. If the metric tensor is not diagonal, the Ricci tensor R_{ab} has millions of terms, and so General Relativity equations cannot be solved analytically.

The Einstein Gravitational Field Equation

The equations for calculating the tensors described above are all specified by the Ricci-Riemann calculus of curved space, expressed in four dimensions. They are not constrained by General Relativity theory. To apply the Ricci-Riemann calculus to General Relativity, Einstein postulated his ***gravitational field equation***, which is

$$G_a{}^b = -8\pi T_a{}^b$$

The tensor $T_a{}^b$ is called the ***energy-momentum tensor***, which describes the characteristics of energy and matter. As explained in *Addendum* document *5,C Calculation of Energy-Momentum Tensor*, Einstein specified a process for calculating the elements of the contravariant form of the energy-momentum tensor T^{ab} from the characteristics of energy and matter for any specified physical model. If the metric tensor is known, one can readily convert the contravariant form T^{ab} of the energy-momentum tensor to the mixed form $T_a{}^b$. The Einstein tensor $G_a{}^b$ is an array of 16 elements denoted $G_1{}^1$, $G_1{}^2$, $G_1{}^3$, etc, and the energy-momentum tensor $T_a{}^b$ is a similar array of elements denoted $T_1{}^1$, $T_1{}^2$, $T_1{}^3$, etc. Hence the gravitational field equation represents 16 separate equations of the form:

$G_1^{\ 1} = -8\pi\, T_1^{\ 1}$, $G_1^{\ 2} = -8\pi\, T_1^{\ 2}$, $G_1^{\ 3} = -8\pi\, T_1^{\ 3}$, etc.

Solving General Relativity Equations Backward

Let us consider the steps for calculating the contravariant energy-momentum tensor T^{ab} from the covariant metric tensor g_{ab}. The Ricci-Riemann calculus of curved space has formulas for making the following calculations. The elements of the contravariant metric tensor g^{ab} are calculated from those of the covariant metric tensor g_{ab}. The elements of the covariant Ricci tensor R_{ab} are computed from the elements of the covariant and contravariant metric tensors g_{ab} and g^{ab}. The covariant Ricci tensor R_{ab} is converted to its mixed form $R_a^{\ b}$ using formulas that employ the covariant and contravariant metric tensors. The mixed Einstein tensor $G_a^{\ b}$ is computed from the mixed Ricci tensor $R_a^{\ b}$ by the following equations: ($G_a^{\ b} = R_a^{\ b}$) for non-diagonal elements, and ($G_a^{\ b} = R_a^{\ b} - ½\, R$) for diagonal elements, where R is the sum of the diagonal elements of the mixed Ricci tensor $R_a^{\ b}$.

The Einstein gravitational field equation represents a set of 16 equations of the form: ($G_1^{\ 2} = -8\pi T_1^{\ 2}$). From these equations, the elements of the mixed energy-momentum tensor $T_a^{\ b}$ are calculated from the elements of the mixed Einstein tensor $G_a^{\ b}$. Finally, the mixed energy-momentum tensor $T_a^{\ b}$ is converted to its contravariant form T^{ab} by means of formulas using the covariant and contravariant metric tensors.

Addendum document *5,C Calculation of Energy-Momentum Tensor* shows how the contravariant energy-momentum tensor T^{ab} is calculated from the characteristics of matter and energy for the particular physical model that is being studied. After the elements of the contravariant metric tensor T^{ab} are obtained, one must solve the equations discussed above in a ***backward*** direction, to find the covariant metric tensor g_{ab} that yields the calculated contravariant energy-momentum tensor T^{ab}. But how does one solve these extremely complicated equations ***backward***? It isn't easy.

Einstein was unable to solve his equations exactly; he could only obtain approximate solutions. The first exact solution was obtained by Karl Schwartzschild, who was cooperating with Einstein. Schwartzschild applied the Einstein equations to a simple model of a star, which can be our sun. In 1916, Einstein published the famous Schwartzschild analysis along with his theory. Sadly, Schwartzschild contracted a rare disease and died before his analysis was published. He was in the German army on the Russian front during World War I.

The Schwartzschild analysis is presented in the *Addendum* documents *5,2 Schwartzschild and Isotropic Solutions of the Einstein Theory* and *5,C Calculation of Energy-Momentum Tensor*. Schwartzschild modeled the star as an ideal fluid having a constant density of matter and no viscosity. Document *5,C* shows how Schwartzschild derived the energy-momentum tensor for his model. The basic General Relativity equations yield the contravariant form of the energy-momentum tensor T^{ab}. To convert this contravariant tensor T^{ab} to its mixed form $T_a^{\ b}$, one normally needs to know the metric tensor, but the metric tensor is not known at this point. The brilliant Schwartzschild calculated the mixed form $T_a^{\ b}$ in a skillful analysis without actually knowing the metric tensor. From the mixed energy-momentum tensor, the gravitational field equation was applied to calculate the mixed Einstein tensor $G_a^{\ b}$.

Schwartzschild assumed that the metric tensor was diagonal, and he specified general expressions with unknown parameters to describe the four diagonal elements of the covariant metric tensor g_{ab}. He applied the equations discussed above to compute equations for the elements of the mixed Einstein tensor $G_a^{\ b}$. He compared these with the elements of $G_a^{\ b}$ computed from his energy-momentum tensor. This allowed him to calculate the unknown parameters of his metric tensor expressions, thereby yielding his final metric tensor solution.

Verification of General Relativity

Based on the Schwartzschild solution, Einstein devised the following three tests to verify his General Relativity theory.

(1) When a light ray passes close to the sun, it should be deflected by 1.8 arc seconds.

(2) A gravitational field causes a clock to run slower, and therefore causes the excited elements on the surface of the sun to oscillate at lower frequency, thereby generating spectra of longer wavelength. The gravitational field of our sun should cause the spectra of light from the sun surface to shift toward the red end of the spectrum by 2.1 parts per million of wavelength.

(3) The planet Mercury has a highly elliptical orbit. The axis of the Mercury orbit advances (or rotates) by 1.39 arc seconds per orbit. Of this advance of the orbit axis, 1.29 arc seconds can be explained with Newton's laws by considering the gravitational attraction of other planets. A residual error of 0.10 arc second per orbit remained, which was explained by the Einstein General theory of Relativity.

These three tests were implemented, and the results established the validity of the Einstein General theory of Relativity.

These measurable effects of General Relativity are tiny: an advance of only 0.10 arc second per orbit of Mercury; a 1.8 arc second deflection of a light beam passing close to the sun, and a gravitational redshift of only 2.1 parts per million in light emitted from the sun. Hence one might wonder why Einstein worked so hard to achieve his theory, and why General Relativity is so highly regarded. ***The answer is that this generalization was essential to provide a solid theoretical foundation for the relativity principle embodied in Special Relativity, which is the original basic version of relativity published in 1905.***

When the predictions of General Relativity were verified, Einstein achieved great fame. After that time, Einstein did little with his General Relativity theory. Special Relativity is very much easier to apply, and has wide applicability. During Einstein's lifetime, General Relativity, with its very complicated tensor analyses, served primarily as a theoretical foundation for justifying Special Relativity.

Computer Solutions of General Relativity

General Relativity equations can only be solved analytically for very simple physical models that yield diagonal metric tensors. In the mid 1960's, powerful computers became readily available, and scientists began to apply them to solve the General Relativity equations for more complicated physical models. The first of these computer studies was performed by Roger Penrose, who analyzed a model of the Big Bang, and this was followed by Stephen Hawking, an associate of Penrose, who analyzed the Black Hole. Other scientists joined in this endeavor, and since then the computer studies of General Relativity have become too numerous to count.

The following appears to be the general approach usually employed in a computer study of General Relativity. The General Relativity equations are inserted into the computer along with algorithms for implementing them numerically. The elements of the metric tensor are expressed numerically in a form having variable coefficients. From this metric tensor information, the computer calculates numerical expressions for the resultant elements of the approximate contravariant energy-momentum tensor (T^{ab}). From the physical model being studied, the computer calculates numerical expressions for the actual energy-momentum tensor (T^{ab}). The elements of the actual and approximate (T^{ab}) energy-momentum tensors are compared, and the differences are used to vary the coefficients of the metric tensor elements in such a way as to minimize the differences. After many iterations, the actual and approximate energy-momentum tensors match one another, and the resultant elements of the metric tensor give the desired result.

It is extremely difficult to make an iterative computation process like this converge to an accurate solution. Consequently, highly sophisticated computational techniques are required to achieve this end.

When Einstein presented his General theory of Relativity, its great mathematical complexity led the public to believe that only a genius could understand the Einstein theory. This belief, which I call the ***Einstein myth***, was apparently encouraged by Einstein and his supporters, because it augmented public support for General Relativity, which certainly deserved all of the credit it received.

The Einstein myth is widely believed today, and so scientists performing computer studies of General Relativity are often treated with awe. They obviously must be geniuses to understand and perform research using Einstein's mysterious General Relativity theory. The Einstein myth has lulled the public into accepting cosmology concepts derived from General Relativity that are an insult to common sense.

There is no question that scientists implementing computer studies of General Relativity are solidly competent, because it is extremely difficult to solve these equations on a computer. However, these technical skills do not place the scientists on a par with Albert Einstein. As the preceding discussion has shown, the General Relativity equations are all precisely specified. No theoretical issues are involved in the application of these equations. Hence, it is nonsense to claim that a scientist performing this computer research is a “genius like Einstein”.

It is essential that visitors to this site not be confused by the Einstein myth, because this can keep them from understanding relativity principles. An understanding of these principles is essential for a productive exploration of the mysteries of cosmology and creation.

Appendix A: The Physical Meaning of a Tensor

The Vector

To explain the tensor, let us start by considering the simpler concept of a vector. Many physical quantities have direction as well as amplitude, and are represented by arrows that are called "vectors". In Fig. 7-2 a vector V is represented in three-dimensional space in terms of its three components in the east, north, and vertical directions. The east, north and vertical directions are often denoted in general form as x, y, and z, and are called “rectangular coordinates”. The components of the vector V along the east (x), north (y), and (z) axes are denoted V_x, V_y, and V_z. The vector V might represent, for example, the position of a body, or its velocity, or its acceleration.

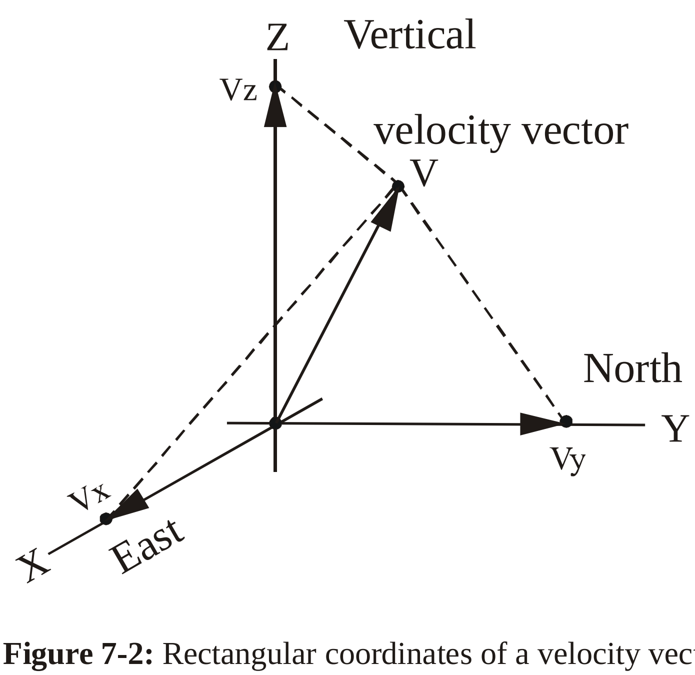

Figure 7-2: Rectangular coordinates of a velocity vector

Let us assume that the vector V in Fig. 7-2 represents the velocity of an aircraft. The vector V shows the direction in which an aircraft is moving, and the length of the vector is proportional to the speed of the aircraft. Three dashed lines are drawn from the tip of the velocity vector V perpendicularly to the three axes. The points where these dashed lines intersect the three axes give the x, y, z components of the vector V, which are the velocity components in the east, north, and vertical directions.

The intersection of the three axes is called the origin. The three components of the vector are measured from the origin along the axes to the intersection points of the three dashed lines. The components of the vector V along the x, y, z axes are denoted V_x, V_y, V_z, where V_x is the aircraft velocity in the easterly (x) direction, V_y is the aircraft velocity in the northerly (y) direction, and V_z is the aircraft velocity in the vertical (z) direction.

The Tensor

The *tensor* can be explained by considering the forces acting inside a mechanical body. A force exerted within a body is specified in terms of ***stress***, which is the force applied per unit area. Two directions are required to describe a stress. One direction gives the direction of the force, and the second direction gives the orientation of the surface to which the force is applied. The stresses exerted within a body are specified in terms of a ***tensor***.

This is illustrated in Fig. 7-3, which shows the forces applied to a cube of material within a body. Each face of the cube is assumed to have unit area. ***Force-per-unit-area*** is called ***stress***, and so the forces applied to the unit-area faces of the cube are stresses. The symbol (p) is used to denote a stress, because pressure is a typical example of a stress.

The stresses in Fig. 7-3 are expressed in term of the x, y, z axes. These stresses are denoted in the form p_{ab}, where the subscript indices a, b can each represent x, y, or z. The first index (a) describes the direction of the force, and the second index (b) describes the orientation of the face to which the force is applied. The orientation of a face is defined by a vector that is perpendicular to the face.

Diagram (a) shows the three ***compression stresses*** that are applied at three faces of the cube, which are denoted p_{xx}, p_{yy}, and p_{zz}. Stress p_{xx} is exerted on the cube in the x-direction at a face that is perpendicular to the x-direction. Stresses p_{yy} and p_{zz} are defined in a similar manner.

The three faces of the cube, which are specified by the second subscript index, are labeled the *x-face*, the *y-face*, and the *z-face*. The x-face is perpendicular to the x-direction, the y-face is perpendicular to the y-direction, and the z-face is perpendicular to the z-direction.

Diagram (b) of Fig 7-3 shows the six ***shear stresses***, which are applied parallel to the cube faces. The two shear stresses applied to the x-face are denoted p_{yx} and p_{zx}. Stress p_{yx} is applied to the x-face in the y-direction, and stress p_{zx} is applied to the x-face in the z-direction. Similarly, the two shear stresses applied to the y-face are denoted p_{xy} and p_{zy}, and the two shear stresses applied to the z-face are denoted p_{xz} and p_{yz}.

Therefore nine separate stresses are required to describe the internal forces within a mechanical body. These stresses are arranged as follows in a 3-by-3 array, which is called a *matrix*

$$\begin{vmatrix} p_{xx} & p_{xy} & p_{xz} \\ p_{yx} & p_{yy} & p_{yz} \\ p_{zx} & p_{zy} & p_{zz} \end{vmatrix}$$

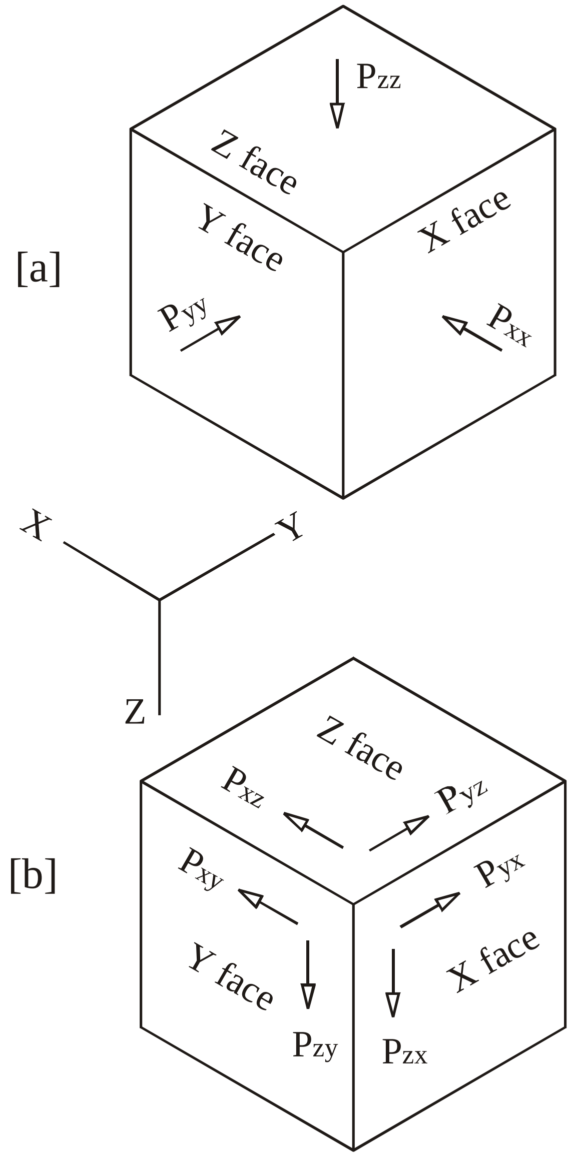

Figure 7-3: Internal stresses within a mechanical body:
[a] compression stresses; [b] shear stresses.

Elements are arranged in this matrix such that the first index indicates the row, and the second index indicates the column. For example, in the first row the first index is x; in the second row the first index is y; and in the third row the first index is z.

This matrix is defined by the variable p_{ab}, which is called a *tensor*. This variable p_{ab} represents the nine stresses in a general form. The subscripts a and b are called indices, and each index can represent x, y, or z.

In summary, a tensor with two indices is required to define a stress, because stress has two independent directions. The first index specifies the direction of the force, and the second index specifies the orientation of the surface to which the force is applied. A surface is perpendicular to the direction specified by the second index.

It is often convenient to describe the x, y, z coordinates by numbers, where x is called x_1, y is called x_2, and z is called x_3. Hence the x index of a stress component is replaced by 1, the y index is replaced by 2, and the z index is replaced by 3. In this form the stress tensor matrix is expressed as

$$\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}$$

Tensors in Relativity Theory. In Relativity theory, the space and time variables cannot be considered separately. Reality is described by *events*, where an event is specified by three spatial coordinates plus a time coordinate, and so has four dimensions. In Relativity theory a vector has 4 elements, and a tensor has 4x4 or 16 elements.

In a Relativity tensor, the x, y, z spatial coordinates are specified as x_1, x_2, x_3. Einstein denoted the time coordinate as x_4, but the author uses the alternative convention where the time coordinate is denoted x_0. Hence a Relativity tensor is represented by a 4-by-4 array of the following form, where the first row and column give the tensor elements associated with time:

$$\begin{vmatrix} p_{00} & p_{01} & p_{02} & p_{03} \\ p_{10} & p_{11} & p_{12} & p_{13} \\ p_{20} & p_{21} & p_{22} & p_{23} \\ p_{30} & p_{31} & p_{32} & p_{33} \end{vmatrix}$$

References

[1] Adrian Bjornson, *A Universe that We Can Believe*, Addison Press, Woburn, MA, 2000, ISBN 09703231-0-7.

[5] Tullio Levi-Civita, *The Absolute Differential Calculus*, 1977, Dover Pub. NY, ISBN 0-486-63401-9.