1,3 The Yilmaz Relativistic Theory of Gravity

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Huseyin Yilmaz developed his gravitational theory while performing PhD research at the Massachusetts Institute of Technology in the early 1950's. In a project on General Relativity, he examined Einstein's approximate analysis of the wavelength shift of light produced by gravity, and discovered that he could solve this problem exactly. With this exact solution he calculated an exact formula for the time component of the metric tensor, g_{00}, and from this he calculated the full metric tensor. This article gives a simplified version of that analysis.

Einstein proved that when light rises in a gravitational field, its wavelength increases, and the light spectrum is shifted toward the red, displaying what astronomers call a ***redshift***. Our analysis starts with Einstein's approximate calculation of the redshift produced by gravity.

Einstein's Approximate Calculation of Gravitational Redshift

To calculate the relativistic effects of gravity, Einstein postulated his ***Principle of Equivalence***, which states that the effects of acceleration and gravity are identical. He analyzed the two elevators shown in Fig. A-1. Elevator (b) is fixed on earth. Elevator (a) is located in free space where there is no gravitational field, and is pulled upward with a constant acceleration equal to the acceleration of gravity (g) experienced on earth. Today it is simpler to assume that elevator (a) is being pushed up with a rocket to achieve a one-G acceleration. A person in elevator (a) would experience an acceleration force on his body that is equal to his gravitational weight when located in elevator (b). ***Einstein's Principle of Equivalence*** states that the effects of acceleration experienced in (a) are identical to those of gravity experienced in (b).

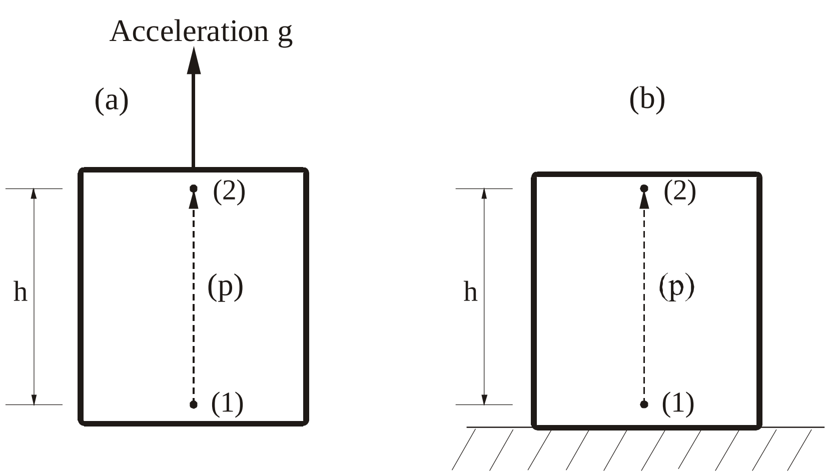

Figure A-1: Elevator (a) is accelerating in free space; elevator (b) is fixed on earth. Light travels from point (1) to point (2).

Suppose a light pulse (p) is emitted from the floor of elevator (a) at point (1) and is received at the ceiling at point (2). During the propagation time of the light, the velocity of the elevator increases. Hence the upward velocity of the receiver at point (2) is greater than that of the emitter at point (1) as far as the light pulse is concerned. Point (2) appears to be moving away from point (1), and so the light spectrum is shifted toward the red as the light moves from point (1) to point (2). Since the effects in elevators (a) and (b) are the same, the gravitational field in elevator (b) must also produce a spectral redshift as light propagates from floor to ceiling.

The time interval (Δt) for the light pulse to propagate from floor to ceiling is approximately (h/c), where (h) is the height of the ceiling above the floor, and (c) is the speed of light. The Greek symbol (Δ), which is called "delta", is commonly used to denote a difference, and so (Δt) represents a time difference. During the propagation time of the light, the velocity (V) of elevator (a) increases by an amount (ΔV) equal to the time interval (Δt) multiplied by the acceleration (g) of the elevator. Hence this velocity increase (ΔV) is approximately equal to

$$\Delta V = g\Delta t = gh/c \tag{1}$$

If there were no acceleration or gravity, and the elevator ceiling were separated from the floor and allowed to move upward with a velocity (ΔV) relative to the floor, the light would be Doppler shifted to a longer wavelength as it propagates from floor to ceiling. Denote the initial wavelength as (L) and the increase in wavelength as (ΔL), so that ($\Delta L/L$) is the fractional increase in wavelength. ***The Doppler formula states that the fractional increase in wavelength ($\Delta L/L$) is approximately equal to ($\Delta V/c$), where c is the speed of light.*** (Instead of L to denote wavelength, scientists generally use λ, called "lambda", which is the lower-case Greek equivalent of the Roman letter L.)

In the accelerating elevator (a), the ceiling ***appears*** to move upward relative the floor with a velocity (ΔV) approximately equal to (gh/c), and this causes the light wavelength to increase. The fractional increase in wavelength ($\Delta L/L$) is approximately equal to ($\Delta V/c$), and so (by Eq. 1) is approximately equal to

$$\Delta L/L = \Delta V/c = gh/c^2 \tag{2}$$

Elevator (b) is fixed on the earth, which has a gravitational field with an acceleration of gravity (g) equal to the upward acceleration of elevator (a). Einstein's ***Principle of Equivalence*** states that gravity and acceleration have exactly equivalent effects. Hence the fixed elevator (b) on earth should produce the same wavelength change as the accelerating elevator (a) located in free space. Consequently, the wavelength (L) of light in the fixed elevator (b) should increase by a fractional amount ($\Delta L/L$) approximately equal to (gh/c^2), as light propagates from floor to ceiling.

Let us consider the meaning of the acceleration of gravity (g). The acceleration of gravity (g) on the surface of the earth is 9.8 meter/sec per second. Let us approximate this as 10 meter/sec per second. When a body is allowed to fall in the earth's gravitational field, its velocity accelerates to 10 meter/sec in one second, to 20 meters/sec in two seconds, to 30 meter/sec in three seconds, etc. During every second the velocity of the body increases by 10 meter/sec. This assumes that air resistance is negligible.

Result for Ideal Earth. If we approximate the earth as an ideal spherically symmetric body, the acceleration of gravity (g) at any point on or above the surface of the earth is

$$g = GM/r^2 \qquad (3)$$

where (r) is distance from the center of the earth, (M) is the mass of the earth, and (G) is the gravitational constant in Newton's law of gravitational force.

To simplify gravitational computations, it is convenient to define a ***normalized mass (m)***, which is related as follows to the actual mass M

$$m = GM/c^2 \qquad (4)$$

The normalized mass m of our sun is (1.475 km). Do not be concerned that normalized mass has units of distance; this has no physical significance. In astronomy calculations, mass is commonly expressed in terms of the mass of the sun, and so the normalized mass of any body is equal to (1.475 km), multiplied by the ratio of the body's mass to the sun's mass. Since the mass of the sun is 333,000 times the mass of the earth, the normalized mass of the earth is (1.475 km) divided by 333,000, which is 4.43 millimeter. Combining Eqs 3, 4 gives the following for the acceleration of gravity (g) expressed in terms of normalized mass:

$$g = mc^2/r^2 \qquad (5)$$

Equation 2 showed that, within the fixed elevator (b) on the surface of the earth, the fractional increase in wavelength ($\Delta L/L$) for light propagating from floor to ceiling is approximately equal to (gh/c^2), and so is approximately equal to:

$$\Delta L/L = g(h/c^2) = (mc^2/r^2)(h/c^2) = mh/r^2 \qquad (6)$$

On the surface of the earth, (r) is the earth radius (6,380 km), (m) is the normalized mass of the earth (4.43 mm), and (h) is the elevator height. The height (h) can be expressed as (Δr), which represents an increase in the distance from the center of the earth. Hence Eq. 6 becomes

$$\Delta L/L = mh/r^2 = m(\Delta r/r^2) \qquad (7)$$

The variable (r) is the distance of the floor from the center of the earth, and (Δr) is the distance from floor to ceiling, which is the height (h) of the elevator. This equation gives Einstein's approximate calculation of gravitational redshift, expressed in terms of the normalized mass (m) of an ideal spherically symmetric earth.

Yilmaz Extension of Einstein Approximate Analysis

Yilmaz found that he could extend Einstein's approximate analysis of gravitational redshift to achieve an exact solution. The following is a simplification of the Yilmaz analysis, derived by the author. (The original Yilmaz analysis was presented in *Story* [2], Appendix E.)

Yilmaz approved of this simplification, because it demonstrates the fundamental nature of the Yilmaz theory, showing that the Yilmaz theory is a direct refinement of the Einstein theory.

The approximate gravitational-redshift formula in Eq. 7 can be converted into an exact formula by reducing the finite changes of wavelength and radial distance (ΔL, Δr) to the infinitesimal changes (dL, dr). The expressions (dL, dr) are called the derivatives of the wavelength (L) and radial distance (r). This is the standard approach used in differential calculus. The ***approximation*** of Eq. 7 yields the following ***exact*** formula for gravitational redshift:

$$dL/L = m(dr/r^2) \qquad (8)$$

Those readers familiar with calculus can refer to Appendix A of this document, which shows that the solution to this differential equation is

$$L_{inf}/L = \exp(m/r) = e^{m/r} \qquad (9)$$

Variable (L) is the wavelength of light at a distance (r) from the center of a body of normalized mass (m), and (L_{inf}) is the wavelength in free space, where (r) is infinite and there is no gravitational field. The expression exp(m/r) is called the exponential of (m/r), and ($e^{m/r}$) is an alternate form for representing this function. The quantity (e) is a constant that is approximately equal to 2.718, and so e^x represents 2.718^x, which is 2.718 raised to the (x) power. The wavelength ratio (L_{inf}/L) is equal to $2.718^{(m/r)}$, which is 2.718 raised to the (m/r) power.

Suppose a clock is synchronized to a light frequency, so that it has one tick for every period of the light wave. The period of the clock (T) would be proportional to the wavelength of the light (L). Hence the clock period would increase when the clock is raised in a gravitational field. We can therefore express Eq. 9 as follows in terms of clock period (T):

$$T_{inf}/T = L_{inf}/L = e^{m/r} \qquad (10)$$

The variable (T_{inf}) is the period of any clock in free space, where there is no gravitational field, and (T) is the period of that same clock when it is moved to a distance (r) from the center of a spherically symmetric body with a normalized mass (m). This equation shows how a gravitational field affects a time measurement. Document *1,7 Metric Equation* of this website shows how relativistic characteristics are calculated from the metric tensor. Equation 7 of that document gives the following:

$$T_{ap}/T = 1/\sqrt{[g_{00}]} \qquad (11)$$

Variable (T) is the clock period measured by an observer within the gravitational field at a point characterized by the value (g_{00}), and (T_{ap}) is the apparent value of this clock period that would be measured at a distant point where the gravitational field is zero. The variable (T_{ap}) is equivalent to (T_{inf}) in Eq. 10. Combining Eqs. 10, 11 gives

$$1/\sqrt{[g_{00}]} = T_{ap}/T = T_{inf}/T = e^{m/r} \qquad (12)$$

Equation 12 yields the following formula for the metric tensor element (g_{00})

$$g_{00} = (1/e^{m/r})^2 = e^{-2m/r} \tag{13}$$

For any variable (x), the expression (e^{-x}) is equal to ($1/e^{x}$). The above analysis shows in principle how Yilmaz calculated the temporal (time) element (g_{00}) of the metric tensor.

Calculating Spatial Elements of the Metric Tensor

Having calculated the temporal element of the metric tensor, Yilmaz calculated the spatial elements by making the following postulate:

> ***The speed of light measured locally in a gravitational field is independent of the direction of the light.***

It should be intuitively obvious that this postulate requires equal values for the three spatial element of the metric tensor (when they are expressed in normal rectangular coordinates). Thus,

$$g_{33} = g_{22} = g_{11} \tag{14}$$

Appendix C of this document presents an analysis showing how the spatial elements of the Yilmaz metric tensor are derived from the temporal element. The analysis shows that the above postulate requires the product ($g_{00}g_{11}$) to be equal to (-1). Therefore g_{11} is equal to

$$g_{11} = - 1/g_{00} = - (1/e^{-2m/r}) = - e^{2m/r} \tag{15}$$

By Eq. 14, the other spatial elements of the metric tensor are

$$g_{33} = g_{22} = g_{11} = - e^{2m/r} \tag{16}$$

Equations 13 and 16 give the metric tensor elements for the gravitational field of a single spherically symmetric body. To account for multiple bodies, the metric tensor is expressed in terms of the gravitational potential, which is denoted (ϕ). Consider a group of spherically symmetric bodies with the normalized masses m_1, m_2, m_3, etc. The gravitational potential (ϕ) at any point outside all of these bodies is equal to

$$\phi = (m_1/r_1) + (m_2/r_2) + (m_3/r_3) + \text{etc.} \tag{17}$$

Variable (r_1) is the distance from the specified point to the center of the body of mass (m_1), (r_2) is the distance from the point to the center of the body of mass (m_2), etc. In terms of the gravitational potential (ϕ) the elements of the metric tensor are

$$g_{00} = \exp(-2\phi) \ , \quad g_{11} = g_{22} = g_{33} = - \exp(2\phi) \tag{18}$$

The Yilmaz Gravitational Field Equation

There are precise formulas for calculating the Einstein tensor $G_a^{\ b}$ from the elements of the metric tensor, g_{00}, g_{11}, g_{22}, g_{33}. Since the Yilmaz metric tensor is diagonal, one can use the formulas calculated by Prof. Herbert Dingle, which are given by this website in *5,B Addendum Appendix B*, section 5. Applying the Dingle formulas to the Yilmaz metric tensor given in Eq. 18 shows that the Yilmaz theory yields a gravitational field equation of the form:

$$- \tfrac{1}{2} G_a^{\ b} = \tau_a^{\ b} + t_a^{\ b} \qquad (19)$$

Yilmaz called ($\tau_a^{\ b}$) the ***stress-energy tensor for matter***, and ($t_a^{\ b}$) the ***stress-energy tensor for the gravitational field***. Except for a factor of (4π), the Yilmaz tensor for matter ($\tau_a^{\ b}$) is similar to the energy-momentum tensor ($T_a^{\ b}$) of the Einstein theory, but is not exactly the same. The Einstein gravitational field equation does not have a tensor corresponding to the Yilmaz tensor for the gravitational field ($t_a^{\ b}$). Einstein tried to derive such a tensor, in order to specify the energy components of the gravitational field, but was unsuccessful. This issue is discussed by this website in *1,6 Limitations of Einstein Gravitational Field Equation*.

The equations specified by Yilmaz for his gravitational field equation are shown in Appendix B of this document, where they are reduced to simpler forms. Since the Yilmaz theory has general formulas for the elements of the metric tensor, the Yilmaz gravitational field equation is generally not needed when the Yilmaz theory is applied. Nevertheless, the Yilmaz gravitational field equation is essential to give the Yilmaz theory a solid theoretical foundation.

General Time-Varying Yilmaz Theory

This basic Yilmaz theory was published by the *Physical Review* in 1958 [5]. This theory is a static solution that applies exactly only when the gravitational field does not change with time. In 1973 Yilmaz generalized his theory to obtain his much more complicated time-varying theory [5], which is presented by this website in *5,5 Addendum Chapter 5* and *5,F Addendum Appendix F*. The time-varying Yilmaz theory has proven that the simple static Yilmaz theory described above gives a very accurate approximation provided that the gravitational field varies slowly relative to the speed of light, a condition that is satisfied in nearly all practical applications. About the only case where the general time-varying solution is needed is the analysis of gravitational waves. From his time-varying theory, Yilmaz has derived general equations for characterizing gravitational waves.

In the time-varying Yilmaz theory, the gravitational potential (ϕ) is generalized to form the gravitational potential tensor, denoted ($\phi_a^{\ b}$). The gravitational potential (ϕ) is the sum of the diagonal elements of the gravitational potential tensor ($\phi_a^{\ b}$), which is called the ***trace*** of the tensor. Yilmaz has shown that his time-varying theory reduces to his static theory when the gravitational potential does not vary with time. As with the static theory, the time-varying Yilmaz theory yields solutions for both the metric tensor and the gravitational field equation.

The metric tensor formula for the time-varying Yilmaz theory is expressed in differential form. Consequently, the time-varying metric tensor cannot in general be calculated analytically, but it can readily be solved with a computer. Computer solutions using the Yilmaz time-varying theory are very much easier to perform than computer solutions using the Einstein gravitational field equation. To apply the Yilmaz time-varying theory, one would solve the static solution, and then modify this static solution by incorporating the differential form of the time-varying solution. Such an analysis will demonstrate that the simple static solution gives an accurate approximation in nearly all practical applications.

We should recognize that the famous Schwartzschild solution of the Einstein theory is a static solution. This was modified to form the Isotropic solution, which is also a static solution. The tests that have been performed to validate General Relativity are all based on these static solutions of the Einstein theory. Consequently, the fact that the basic Yilmaz theory is only a static solution is not a strong limitation.

Applying the Yilmaz Theory

With the Einstein theory, the Einstein gravitational field equation is solved to obtain the corresponding metric tensor. This requires that the equations be solved in the ***backward*** direction, which can be an extremely complicated process.

The Yilmaz theory has general formulas for the metric tensor elements, and these are solved directly when the Yilmaz theory is applied to a particular physical model. Consequently, the Yilmaz gravitational field equation is not used in most applications of the Yilmaz theory. The Yilmaz gravitational field equation is only needed to provide a theoretical foundation for the theory. Even when the general time-varying solution is required, the Yilmaz theory still has a general formula for the metric tensor, which is solved by applying it to the physical model being studied. The Yilmaz metric tensor formula for a time-varying gravitational field is in differential form, and so generally requires a computer to solve it. Nevertheless, even with the time-varying theory, the Yilmaz theory is very much easier to apply than the Einstein theory.

How the Yilmaz Theory Was Accepted by the Physical Review

When Yilmaz first submitted his static gravitational theory to the prestigious *Physical Review*, it was rejected. In this paper, Yilmaz presented his static metric tensor and applied Lagrangian analysis to derive the formula for his gravitational field equation, which is given in Eqs. B-1 to B-4 of Appendix B. A reviewer of the paper strongly objected to the claim by Yilmaz that his specified gravitational field equation would be satisfied with his metric tensor. The reviewer stated that the calculation for obtaining the Ricci tensor involves "ten nonlinear coupled partial differential equations, with three variables and numerous singularities". He asked, "What is the chance that the predicted solution will be achieved? This claim is absurd!"

Yilmaz did not know what to do. Since the Yilmaz metric tensor is diagonal, one can use the formulas for calculating the Einstein tensor that were derived by Prof. Herbert Dingle, and are presented by this website in *5,B Addendum Appendix B*, section B,5. From these Dingle formulas, one can theoretically calculate from the elements of the Yilmaz metric tensor (g_{ab}) the corresponding elements of the Einstein tensor ($G_a{}^b$). However, try as he might, Yilmaz was unable to solve the complicated Dingle formulas without making numerous errors.

Yilmaz was working at the Sylvania Applied Research Laboratory, which was located in Waltham, Massachusetts. The Director of this laboratory, Dr. Leonard Sheingold, had faith in Dr. Yilmaz. He assigned this problem to three mathematicians, all women. They were located in separate departments of the Sylvania company and were unaware of each other's work. The mathematicians knew calculus well and were very methodical.

The mathematicians were given the formulas by Prof. Herbert Dingle (presented by this website in *5,B Addendum Appendix B*), along with the formulas for the metric tensor elements of the Yilmaz theory shown in Eq. 18. Yilmaz stated that (ϕ) in these metric tensor equations varies with the spatial coordinates x, y, z, but does not vary with time. After a few weeks, the mathematicians had completed their work. All three had found the same answer, which proved that all elements of ($-½G_a{}^b$) exactly matched the elements of the expression for ($\tau_a{}^b + t_a{}^b$) that Yilmaz had predicted. The "absurd" claim by Yilmaz was correct! Armed with the results from the mathematicians, Yilmaz submitted a revised manuscript, which was rapidly approved by the *Physical Review* and published in 1958. [6]

This anecdote shows that the Yilmaz theory has profound mathematical integrity. This website shows in *5,5 Addendum Chapter 5*, the derivation of the formulas for the stress energy tensors ($\tau_a{}^b$, $t_a{}^b$) for the general time-varying Yilmaz theory. The analysis proves that ($\tau_a{}^b + t_a{}^b$) is equal to ($-½G_\mu{}^\nu$) for all possible applications of the Yilmaz theory. Therefore the Yilmaz gravitational field equation is always satisfied and has a solid theoretical foundation.

Over the centuries, a multitude of ad-hoc postulates have been proposed to explain various physical phenomena. Big Bang studies abound in such postulates. Some ad-hoc postulates might indeed be true, but without supporting evidence one cannot determine what postulates have merit. The Yilmaz theory is in an entirely different category from these ad-hoc postulates. The analysis showing that the Yilmaz gravitational field equation is always exactly satisfied proves that the Yilmaz theory has a solid mathematical foundation. ***The Yilmaz theory should be taken seriously, and we should carefully investigate the predictions of the theory.***

Why Has the Yilmaz Theory Been Ignored?

You may ask, “If the Yilmaz theory is so profound, why has it been largely ignored for a half century?” The answer is clear. Hundreds, and possibly thousands, of scientists have based their profession careers on studies using the Einstein gravitational field equation. If the Yilmaz theory is correct, the Einstein gravitational field equation must be wrong, and this enormous research effort, based on the Einstein gravitational field equation, would become irrelevant.

To claim that the Einstein gravitational field equation is wrong does not mean that Einstein’s General theory of Relativity is wrong. The Yilmaz theory is a refinement of the Einstein theory, which applies the principles established by Einstein. If the Yilmaz theory is correct, General Relativity is basically correct. Einstein did not quite get the correct answer with his gravitational field equation. Einstein admitted that his gravitational field equation was not completely accurate when he recognized in 1945 that his theory would not apply accurately under conditions of very high density of field and matter [7].

Appendix A
Yilmaz Extension of Einstein Approximate Analysis

Yilmaz found that he could extend Einstein's approximate analysis of gravitational redshift to achieve an exact solution. The following is a simplification of the Yilmaz analysis, derived by the author. The original Yilmaz analysis was presented in *Story* [2], Appendix E.

Equation 7 gave the following approximate formula for gravitational redshift derived by Einstein, which was modified to express it in terms of the normalized mass of an ideal spherically symmetric earth:

$$\Delta L/L = mh/r^2 = m(\Delta r/r^2) \qquad \text{(A-1)}$$

Parameter (m) is the normalized mass of the earth (4.43 mm), (r) is the radius of the earth (6,380 km), and (h) is the height of the elevator. The height (h) can be represented as (Δr), which represents an increase in the distance from the center of the earth. The variable (r) is the distance of the floor from the center of the earth, and (Δr) is the distance from floor to ceiling, which is the height (h) of the elevator.

This approximate formula for gravitational redshift can be converted into an exact formula by reducing the finite changes (ΔL, Δr) of wavelength and radial distance to the infinitesimal changes (dL, dr). The expression (dL) is called the derivative of the wavelength (L), and (dr) is called the derivative of the radial distance (r). This is the standard approach used in differential calculus. The resultant exact differential formula for gravitational redshift is:

$$dL/L = m(dr/r^2) \qquad \text{(A-2)}$$

Calculus formulas tell us that (dr/r^2) is equal to -d(1/r), and (dL/L) is equal to d[Ln(L)], where d(1/r) is the derivative of (1/r) and d[Ln(L)] is the derivative of Ln(L), which is the natural logarithm of wavelength (L). Hence Eq. A-2 can be expressed as

$$d[Ln(L)] = - m\, d(1/r) \qquad \text{(A-3)}$$

Integrating this gives

$$Ln(L) = - m(1/r) + C \qquad \text{(A-4)}$$

where C is an unknown constant. Consider the value of the wavelength (L) in free space where the radial distance (r) is infinite and the gravitational field is zero. We denote this wavelength as L_{inf}. With (r) set equal to infinity, the quantity (1/r) is zero, and the equation becomes

$$Ln(L_{inf}) = C \qquad \text{(A-5)}$$

Subtracting Eq. A-4 from Eq. A-5 eliminates the C constant to give

$$\mathrm{Ln}(L_{inf}) - \mathrm{Ln}(L) = (m/r) \qquad \text{(A-6)}$$

The difference between the logarithms of two quantities is the logarithm of the ratio of the quantities. Hence Eq. A-6 becomes

$$\mathrm{Ln}(L_{inf}/L) = (m/r) \qquad \text{(A-7)}$$

Taking the anti-logarithm of this equation gives

$$L_{inf}/L = \exp(m/r) = e^{m/r} \qquad \text{(A-8)}$$

The expression exp(m/r) is called the exponential of (m/r), and ($e^{m/r}$) is an alternate form for representing this function. The quantity (e) is a constant approximately equal to 2.718, and so e^x represents 2.718^x, which is 2.718 raised to the (x) power. The wavelength ratio (L_{inf}/L) is equal to $2.718^{(m/r)}$, which is 2.718 raised to the (m/r) power. Equation A-8 gives the exact expression for gravitational redshift that was derived by Yilmaz. This was the starting point in the derivation of the Yilmaz theory of gravity.

Appendix B
The Yilmaz Gravitational Field Equation

To calculate the Einstein tensor $G_a^{\ b}$ from the elements of the metric tensor, one can use the formulas by Prof. Herbert Dingle, which are given by this website in *5,B Addendum Appendix B*, section B,5. Applying the Dingle formulas to the Yilmaz metric tensor in Eq. 18 shows that the Yilmaz theory yields a gravitational field equation of the form:

$$-\tfrac{1}{2} G_a^{\ b} = \tau_a^{\ b} + t_a^{\ b} \qquad \text{(B-1)}$$

Yilmaz called ($\tau_a^{\ b}$) the ***stress-energy tensor for matter***, and ($t_a^{\ b}$) the ***stress-energy tensor for the gravitational field***. Except for a factor of (4π), the Yilmaz tensor for matter ($\tau_a^{\ b}$) is similar to the energy-momentum tensor ($T_a^{\ b}$) of the Einstein theory, but is not exactly the same. The Einstein gravitational field equation does not have a tensor corresponding to the Yilmaz tensor for the gravitational field ($t_a^{\ b}$). Einstein tried to derive such a tensor, in order to specify the energy components of the gravitational field, but was unsuccessful. This point is discussed by this website in *1,6 Limitations of Einstein Gravitational Field Equation.*

The stress-energy tensor for matter ($\tau_a^{\ b}$) has only one element in the static Yilmaz theory, which is the element for the time dimension ($\tau_0^{\ 0}$), the remaining 15 elements being zero. This temporal element ($\tau_0^{\ 0}$) is equal to

$$\tau_0{}^0 = - e^{-2\phi} \nabla^2\phi \qquad \text{(B-2)}$$

Function ($\nabla^2\phi$) is called the Laplacian of (ϕ) and is given by

$$\nabla^2\phi = (\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial z^2) \qquad \text{(B-3)}$$

The quantity ($\partial\phi/\partial x$) is called the partial derivative of function (ϕ) with respect to (x), and is obtained by differentiating function (ϕ) with respect to dimension (x) when the other three dimensions (y, z, and t) are held constant. Hence the partial derivative $\partial\phi/\partial x$ is the slope of the function (ϕ) measured in the x-direction. The function ($\partial^2\phi/\partial x^2$) is the second derivative of function (ϕ) with respect to x, with the three other dimensions (y, z, and t) held constant, and so ($\partial^2\phi/\partial x^2$) is the rate of change of the slope of function (ϕ) in the x-direction.

The Yilmaz stress-energy tensor for the gravitational field is given as follows by Yilmaz:

$$t_a{}^b = - \partial_a\phi\, \partial^b\phi + ½\, \delta_a{}^b\, \Sigma_i\, \partial_i\phi\, \partial^i\phi \qquad \text{(B-4)}$$

The quantity ($\delta_a{}^b$) is unity for diagonal elements (where a = b), and is zero for non-diagonal elements (where a $\neq$ b). The summation sign (Σ_i) indicates that ($\partial_i\phi\, \partial^i\phi$) is summed over all four values (0, 1, 2, 3) of the index (i). The function ($\partial_a\phi$) is defined as ($\partial\phi/\partial x_a$).

The function ($\partial^b\phi$) is defined as [$\Sigma_i\; g^{bi}(\partial\phi/\partial x_i)$]. Since the Yilmaz metric tensor is diagonal, the metric tensor element (g^{bi}) is zero unless the two indices (b, i) are equal. Hence this summation reduces to a single element, and the function ($\partial^b\phi$) becomes $g^{bb}(\partial\phi/\partial x_b)$, which can be expressed as ($g^{bb}\partial_b\phi$). Hence Eq B-4 can be simplified to

$$t_a{}^b = - g^{bb}\partial_a\phi\, \partial_b\phi + F/2 = - g^{bb}(\partial\phi/\partial x_a)(\partial\phi/\partial x_b) + F/2 \qquad \text{(B-5)}$$

The function F is zero for non-diagonal elements of the stress-energy tensor ($t_a{}^b$) (i.e., for b $\neq$ a), and is given by the following for diagonal elements (for b = a):

$$F = \Sigma_i\, g^{ii}\, (\partial_i\phi)^2 = g^{00}\, (\partial_0\phi)^2 + g^{11}(\partial_1\phi)^2 + g^{22}(\partial_2\phi)^2 + g^{33}(\partial_3\phi)^2 \qquad \text{(B-6)}$$

The elements of the contravariant Yilmaz metric tensor are

$$g^{00} = 1/g_{00} = e^{2\phi}, \qquad g^{33} = g^{22} = g^{11} = 1/g_{11} = - e^{-2\phi} \qquad \text{(B-7)}$$

Hence the function F becomes

$$F = e^{2\phi}\, (\partial\phi/\partial\tau)^2 - e^{-2\phi}\, [(\partial\phi/\partial x)^2 + (\partial\phi/\partial y)^2 + (\partial\phi/\partial z)^2] \qquad \text{(B-8)}$$

Variable (τ) is relativistic time, which is equal to (ct). In summary, the stress-energy tensor for the gravitational field is given by

$$t_a{}^b = - g^{bb}(\partial\phi/\partial x_a)(\partial\phi/\partial x_b) + F/2 \qquad \text{(B-9)}$$

where F is zero for non-diagonal elements of this tensor (for $b \neq a$) and is given by Eq B-8 for diagonal elements. Variable (x_a) represents ($\tau = ct$) for ($a = 0$), it represents (x) for ($a = 1$), it represents (y) for ($a = 2$), and it represents (z) for ($a = 3$). This also holds for variable (x_b). Metric tensor element (g^{bb}) is equal to ($e^{2\phi}$) for ($b = 0$), and is equal to ($- e^{-2\phi}$) for ($b = 1, 2, 3$).

The above shows that the stress-energy tensor for matter ($\tau_a{}^b$) reduces to a single element ($\tau_0{}^0$) given by Eq. B-2, and the stress-energy tensor for the gravitational field ($t_a{}^b$) is given by Eqs B-8 and B-9. These equations are remarkably simple.

Appendix C
Calculating Spatial Elements of Yilmaz Metric Tensor

Equation 12 of the body of this article gives the exact formula derived rigorously by Yilmaz for the time element g_{00} of the metric tensor. To obtain the spatial elements of the metric tensor, Yilmaz postulated: ***The speed of light measured locally in a gravitational field is independent of direction***. This obviously requires an isotropic metric tensor with equal spatial metric-tensor elements in rectangular coordinates:

$$g_{11} = g_{22} = g_{33} \qquad \text{(C-1)}$$

The following analysis by Yilmaz shows that ($g_{00}g_{11}$) must equal (-1).

An electromagnetic wave is specified in four-dimensional space-time coordinates by the requirement that $\Box^2\xi = 0$, where ξ is the vector potential of the electromagnetic field and $\Box^2$ is the d'Alembertian operator. It can be shown that $\Box^2\xi$ is equal to

$$\Box^2\xi = (1/\sqrt{[-\boldsymbol{g}]})\, \partial_\alpha\{\sqrt{[-\boldsymbol{g}]}\, g^{\alpha\beta}\, \partial_\beta\xi\} \qquad \text{(C-2)}$$

where $\boldsymbol{g}$ is the determinant of the metric tensor, and ($-\boldsymbol{g}$) is always positive. Since the indices α, β are repeated, this expression is summed over the four values (0, 1, 2, 3) of these two indices. This equation is more general than electromagnetic theory and can represent any wave propagating at the speed of light. Differentiating Eq. C-2 by parts gives

$$\Box^2\xi = (1/\sqrt{[-\boldsymbol{g}]})(\sqrt{[-\boldsymbol{g}]}g^{\alpha\beta})\, \partial_\alpha\partial_\beta\xi + (1/\sqrt{[-\boldsymbol{g}]})\, \partial_\beta\xi\, \partial_\alpha\{\sqrt{[-\boldsymbol{g}]}g^{\alpha\beta}\} \qquad \text{(C-3)}$$

For a diagonal metric tensor, $g^{\alpha\beta}$ is zero unless $\beta = \alpha$, and so β can be replaced by α. To obtain the wave equation, set $\Box^2\xi$ equal to zero. This gives

$$0 = \Box^2\xi = g^{\alpha\alpha}\, \partial_\alpha{}^2\xi + (1/\sqrt{[-\boldsymbol{g}]})\, \partial_\alpha\xi\, \partial_\alpha\{\sqrt{[-\boldsymbol{g}]}\, g^{\alpha\alpha}\} \qquad \text{(C-4)}$$

The terms are to be summed over the four values (0, 1, 2, 3) of the index α. This summation gives the following wave equation, where the index k represents the 3 spatial indices (1, 2, 3):

$$0 = \Box^2\xi = g^{00}\,\partial_0{}^2\xi + \Sigma_k\, g^{kk}\,\partial_k{}^2\xi + (1/\sqrt{[-\boldsymbol{g}]})\,\partial_0\xi\,\partial_0\{\sqrt{[-\boldsymbol{g}]}\,g^{00}\} + (1/\sqrt{[-\boldsymbol{g}]})\,\Sigma_k\,\partial_k\xi\,\partial_k\{\sqrt{[-\boldsymbol{g}]}\,g^{kk}\} \qquad \text{(C-5)}$$

As specified, the metric tensor does not vary with time. Since ∂_0 represents differentiation relative to time, $\partial_0\{\sqrt{[-\boldsymbol{g}]}\,g^{00}\}$ in the third term must be zero. The last term is proportional to the spatial derivative of the electromagnetic field, $\partial_k\xi$. Since the speed of light does not vary with direction, the wave equation cannot vary with $\partial_k\xi$. Consequently the following factor of the last term must be zero:

$$\partial_k\{\sqrt{[-\boldsymbol{g}]}\,g^{kk}\} = 0 \qquad \text{(C-6)}$$

Since the metric tensor is diagonal, the negative of the determinant $\boldsymbol{g}$ of the metric tensor is

$$-\boldsymbol{g} = -\,g_{00}\,g_{11}\,g_{22}\,g_{33} = -\,g_{00}\,g_{11}(g_{11})^2 \qquad \text{(C-7)}$$

By Eq. C-1, g_{22} and g_{33} were set equal to g_{11}. In Eq. C-6, set g^{kk} equal to g^{11} and apply Eq. C-7 to obtain

$$\partial_1\{\sqrt{[-\boldsymbol{g}]}\,g^{11}\} = \partial_1\{\sqrt{[-g_{00}\,g_{11}]}\,g_{11}g^{11}\} = 0 \qquad \text{(C-8)}$$

Since $g_{11}g^{11}$ is unity for a diagonal metric tensor, this reduces to

$$\partial_1\{\sqrt{[-g_{00}\,g_{11}]}\,\} = 0 \qquad \text{(C-9)}$$

To satisfy this requirement, $(g_{00}\ g_{11})$ must be constant. With no gravitational field, special relativity applies and $-(g_{00}\ g_{11})$ is unity. Hence $-(g_{00}\ g_{11})$ must always be unity. Since $g_{00} = e^{-2\phi}$, g_{11} is equal to:

$$g_{11} = -\,1/g_{00} = -\,1/e^{-2\phi} = -\,e^{2\phi} \qquad \text{(C-10)}$$

By Eq. C-1, the other spatial elements of the metric tensor are

$$g_{33} = g_{22} = g_{11} = -\,e^{2\phi} \qquad \text{(C-11)}$$

References

[1] Adrian Bjornson, *A Universe that We Can Believe*, Addison Press, Woburn, MA, 2000, ISBN 09703231-0-7.

[2] Adrian Bjornson, *The Scientific Story of Creation*, Addison Press, Woburn, MA, 2002, ISBN 09703231-2-3.

[5] Huseyin Yilmaz, "New Approach to Relativity and Gravitation", *Annals of Physics*, Academic Press, NY, 1973, pp. 179-200.

[6] Huseyin Yilmaz, "New Approach to General Relativity", *Physical Review*, vol. 111, No. 5, Sept. 1, 1958, pp 1417-1426,

[7] Albert Einstein, *The Meaning of Relativity*, Princeton University Press, 5th ed., 1953, ISBN 0-691-02352-2, (1st ed. 1921), (*See* appendix for 2nd ed., 1945, p. 129).