1,4 Application of the Yilmaz Theory to Cosmology

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Introduction

Newton's theory of gravity is not an adequate basis for a quantitative study of cosmology. A gravitational theory is required that incorporates relativistic effects. After Einstein presented his General theory of Relativity in 1916, scientists began to apply it to cosmology, and since then it has provided the foundation for nearly all cosmology studies.

In 1945, Einstein [5] recognized that his theory predicts a singularity having essentially infinite density of matter at the instant of the Big Bang. He knew that a singularity is physically impossible, and so he concluded that his General Relativity equations cannot hold accurately under an extremely high density of field and matter. [5] However, some scientists have avoided the Big Bang singularity by assuming a cosmology model that does not start with a Big Bang.

About a decade after Einstein's death, computer studies applied Einstein's gravitational field equation to a massive star having a mass-to-radius ratio exceeding the Schwartzschild limit (236,000 times the ratio for our sun). These studies proved that, if Einstein's equations are correct, such a star ***must*** collapse to form a Black Hole singularity of infinite density. There is extensive evidence of stars with mass-to-radius ratios greatly exceeding the Schwartzschild limit, and so the Black Hole singularity enigma cannot be avoided. If we agree with Einstein that a singularity is physically impossible, we must accept Einstein's 1945 conclusion [5] that his gravitational field equation does not hold accurately for an extremely high density of matter.

This conclusion proves that Einstein's gravitational field equation is not adequate for a study of cosmology. A better relativistic theory of gravity is needed, and the Yilmaz gravitational theory fills that need. The Yilmaz theory is a refinement of the Einstein General theory of Relativity, which satisfies all of the requirements of General Relativity except that it has a different gravitational field equation. Einstein did not derive his gravitational field equation rigorously; it was his best guess, after trying a number of alternatives. In 1999, John A. Peacock reported in his authoritative textbook on cosmology, "The Einstein gravitational field equation cannot be derived in a rigorous sense; all that can be done is to follow Einstein and start by thinking of the simplest form that such an equation might take." [6] (p. 19)

As explained in 1,3 *Yilmaz Relativistic Theory of Gravity*, Yilmaz derived his theory by extending Einstein's approximate calculation of gravitational redshift. This analysis yielded a rigorous formula for the metric tensor, and Yilmaz calculated from this his rigorous gravitational field equation. The Yilmaz gravitational theory has a solid mathematical foundation and therefore provides a scientific basis for studying cosmology.

The Yilmaz Cosmology Model

In the first paper on his gravitational theory, Yilmaz [7] applied his theory to a simple cosmology model. ***He postulated that the universe has a constant average density of matter***

that extends to infinity and does not change with time. The metric tensor elements for this model are calculated in Appendix B, and are:

$$g_{00} = \exp[-(r/r_0)^2] ; \quad g_{11} = g_{22} = g_{33} = - \exp[(r/r_0)^2] \qquad (1)$$

The parameter (r) is distance from the observer, and (r_0) is a constant that is related as follows to the average mass density of the universe, denoted (ρ):

$$r_0^2 = 3c^2/4\pi\rho G \qquad (2)$$

Parameter G is the gravitational constant for Newton's law of gravity. We will see that (r_0) is related to the Hubble expansion of the universe, and is equal to the distance at which the Hubble Law predicts a galaxy should recede at the speed of light. Recent data indicates a Hubble constant of 20 km/sec per million light years of galaxy distance. If the expansion of the universe were constant, a galaxy 15 billion years away would recede at the speed of light (300,000 km/sec), and so r_0 is 15 billion light years. This is often considered to be the radius of the observable universe. Appendix B shows that for ($r_0 =$ 15 billion light years), Eq. 2 predicts an average mass density of the universe (ρ) equivalent to 9.6 hydrogen atoms per cubic meter. This is in good agreement with recent astronomical measurements, which give average mass densities equivalent to 3.0 to 7.2 hydrogen atoms per cubic meter.

Yilmaz presented this cosmology model merely as an example to illustrate the use of his gravitational theory, and did not pursue the model after his original paper. Nevertheless, we will see that the simple model yields some remarkable cosmology predictions.

Effect of Gravity on Light Speed, Clock Rate, and Spatial Dimension

The metric equation, which is based on the metric tensor, is explained in document 1,7 *Metric Equation*. From the metric equation, one can calculate the effect of the gravitational field on the speed of light, a spatial dimension, and a clock period. Expressions c_{ap}, Δx_{ap}, and ΔT_{ap} denote the "apparent" (or "coordinate") values of speed of light, spatial dimension, and clock period observed here on earth, and c, Δx, and ΔT are the corresponding "true" (or "proper") values observed at the distant location. Table 3 of 1,7 *Metric Equation* shows that the speed of light ratio (c_{ap}/c) is $\surd[-g_{00}/g_{11}]$, the spatial dimension ratio ($\Delta x_{ap}/\Delta x$) is $1/\surd[-g_{11}]$, and the clock period ratio ($\Delta T_{ap}/\Delta T$) is $1/\surd[g_{00}]$, where (g_{00}) and (g_{11}) are the time and spatial elements of the metric tensor. The spatial metric tensor elements (g_{11}, g_{22}, g_{33}) are assumed to be equal.

In the following three equations, these general formulas are applied to the Yilmaz cosmology model by using the expressions for (g_{00}) and (g_{11}) in Eq. (1):

$$(c_{ap}/c) = \surd[-g_{00}/g_{11}] = \exp[-(r/r_0)^2] \qquad \text{(speed of light)} \qquad (3)$$

$$(\Delta x_{ap}/\Delta x) = 1/\surd[-g_{11}] = \exp[-(r/r_0)^2/2] \qquad \text{(spatial dimension)} \qquad (4)$$

$$(\Delta T/\Delta T_{ap}) = \surd[g_{00}]. = \exp[-(r/r_0)^2/2] \qquad \text{(clock rate)} \qquad (5)$$

The expression ($\Delta T/\Delta T_{ap}$) represents the relative clock rate, which is the reciprocal of the relative clock period ($\Delta T_{ap}/\Delta T$). The ratios in Eqs. 3, 4, 5 are plotted in Fig. 1, where (r_0) is set equal to 15 billion light years and (r) is the true (or proper) distance to the galaxy. Since (r_0) is generally considered to be the approximate radius of the "observable" universe, it is usually assumed that we cannot observe galaxies beyond 15 billion light years. However, in the Yilmaz cosmology model, the observable universe extends far beyond 15 billion light-years. As shown in Fig. 1, the speed of light becomes very small beyond 30 billion light years ($2r_0$); while a spatial dimension and a clock rate become very small beyond 45 billion light years ($3r_0$).

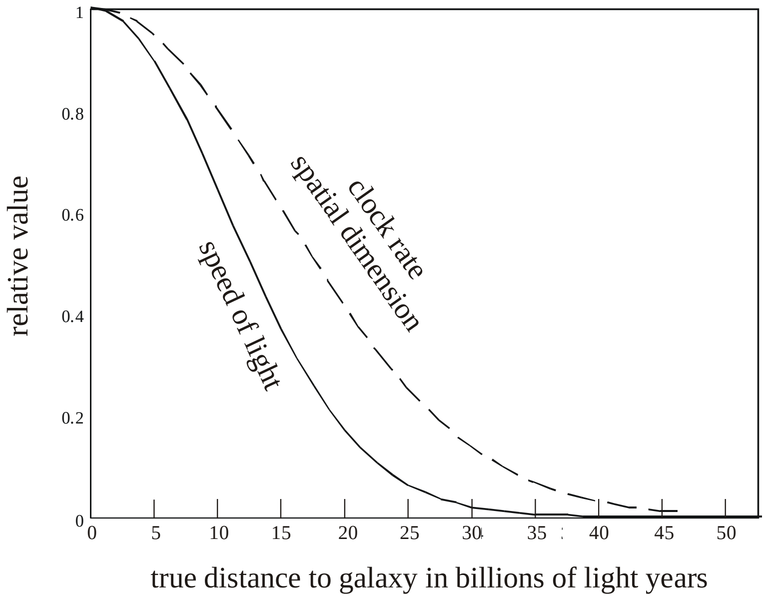

Figure 1: Apparent speed of light (solid), clock rate (dashed), and spatial dimension (dashed), versus distance to a galaxy

Hubble Expansion of the Universe

Newton's gravitational theory states that a body travels in a straight line at constant velocity unless a force is applied to it. When the earth travels around the sun, it follows a curved path because of the gravitational force exerted by the sun on the earth. However, there is no gravitational force in the Einstein and Yilmaz theories. Instead, the gravity of the sun affects the motion of the earth by curving the space around the sun. Since no gravitational force is applied to the earth, the earth follows the equivalent of a straight line in this curved space, which is called a ***geodesic*** path. In normal flat Euclidean space, the shortest distance between two points is a straight line; whereas in curved space the shortest distance between two points is a ***geodesic*** path. To determine the paths of bodies throughout our universe we must calculate the geodesic paths for those bodies. This is calculated from the geodesic equation, which is a fundamental equation of the Ricci-Riemann calculus of curved space.

In Appendix A, the geodesic equation is applied to the Yilmaz cosmology model to calculate the geodesic path followed by a galaxy located at a true distance (r) from the earth. The geodesic equation shows that a galaxy moves in the radial direction away from the earth with an apparent velocity (V_{ap}) given by

$$(V_{ap}/c_{ap})^2 = 1 - \exp[-(r/r_0)^2] \qquad (6)$$

where (c_{ap}) is the apparent speed of light observed at the galaxy location. Equation 6 is plotted in Fig. 2, which shows that the relativistic effect of mass in the universe makes the universe expand. ***The plot shows that the Hubble expansion of the universe is a natural relativistic effect, which does not require a Big Bang explosion. This amazing result was discovered by Yilmaz in 1958! [7]***

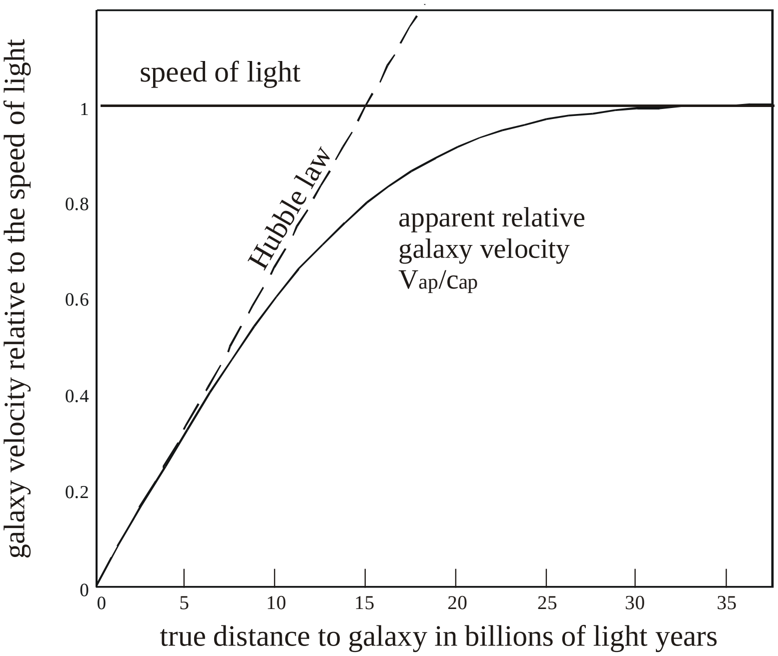

Figure 2: Apparent galaxy velocity relative to apparent speed of light, compared with Hubble law.

The dashed line in Fig 2 shows the “Hubble Law”, which assumes that the Hubble expansion rate remains constant throughout the universe. For a Hubble constant of 20 km/sec per million light years of galaxy distance, galaxy velocity would ideally reach the speed of light (300,000 km/sec) at a distance (r_0) of 15,000 million light years, or 15 billion light years. The solid curve shows the actual plot of the apparent galaxy velocity. This velocity initially follows the straight-line plot of the Hubble Law, but at very large distances it departs drastically from the Hubble Law. The apparent velocity approaches the speed of light gradually, but never quite reaches it.

Figure 2 shows the ratio of the apparent galaxy velocity relative to the apparent speed of light. This is the velocity ratio that would be calculated by applying the Doppler formula to the redshift of the spectrum received from the galaxy.

Figure 2 shows the major component of the apparent galaxy velocity, which is caused by the average distribution of matter throughout the universe. In addition to this, the galaxy has smaller velocity components (in both radial and tangential directions) that are caused by the gravitational fields of local bodies. These smaller velocity components are associated with the rotation of galaxies, the motion of galaxies within clusters, the motion of clusters within super-clusters, etc.

The author uses the terms "apparent" and "true" to represent concepts that Einstein described with the terms "coordinate" and "proper". The apparent (or coordinate) velocity is the velocity of a distant galaxy that one would measure with any instrument on earth. An increment of true (or proper) distance is the value that would be measured at the distant galaxy.

Apparent Compression of the Universe

The plot of spatial dimension in Fig 1 shows that a dimension appears to contract with distance. Consequently, the apparent distance to a galaxy is less than the true distance. Figure 3 gives a plot of apparent distance versus true distance. The data were obtained from the *Addendum* document *5,E Apparent Distance in Yilmaz Cosmology Model*, which presents a table giving values of (r_{ap}/r_0) versus (r/r_0). Because of the strong contraction of spatial dimensions at great distance, the maximum apparent distance to a galaxy is finite, even though the model assumes that the true galaxy distance extends to infinity. The maximum apparent distance to a galaxy is $\sqrt{[\pi/2]}$ times the distance r_0 (15 billion light years) derived from the Hubble law. Maximum apparent galaxy distance of the Yilmaz cosmology model is $\sqrt{[\pi/2]}$ times 15 billion light years, which is 18.80 billion light years.

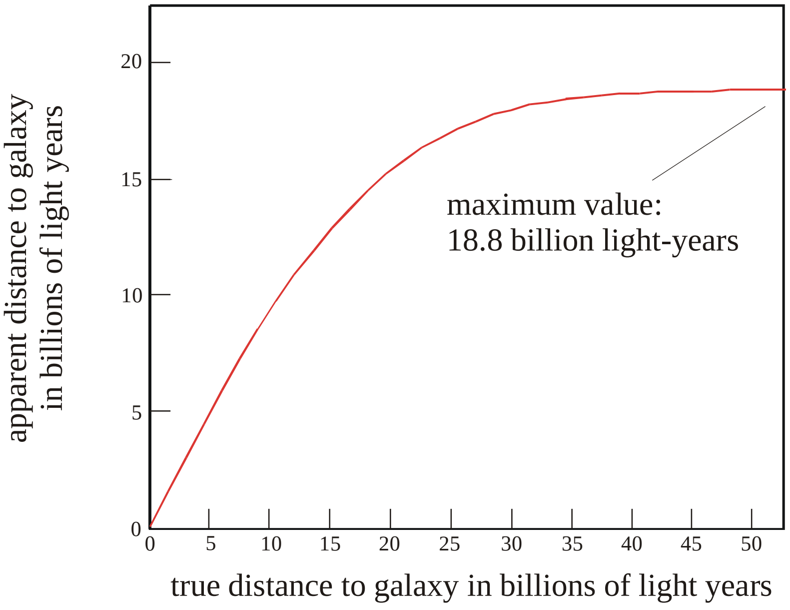

Figure 3: Apparent distance to galaxy verses true distance

For a galaxy close to the apparent limit of 18.8 billion light years, there is strong compression of the apparent size of the galaxy, and so the apparent density of matter becomes very high as this limit is approached. This effect is displayed in Fig. 4, which shows how the apparent density of the universe increases with distance from the earth (the center dot). The inner circle is at an apparent distance of 7.5 billion light years, and the periphery is at an apparent distance of 18.8 billion light years. The universe is quite regular out to 7.5 billion light years, where the relative density is 1.5. At the periphery the density is extremely high.

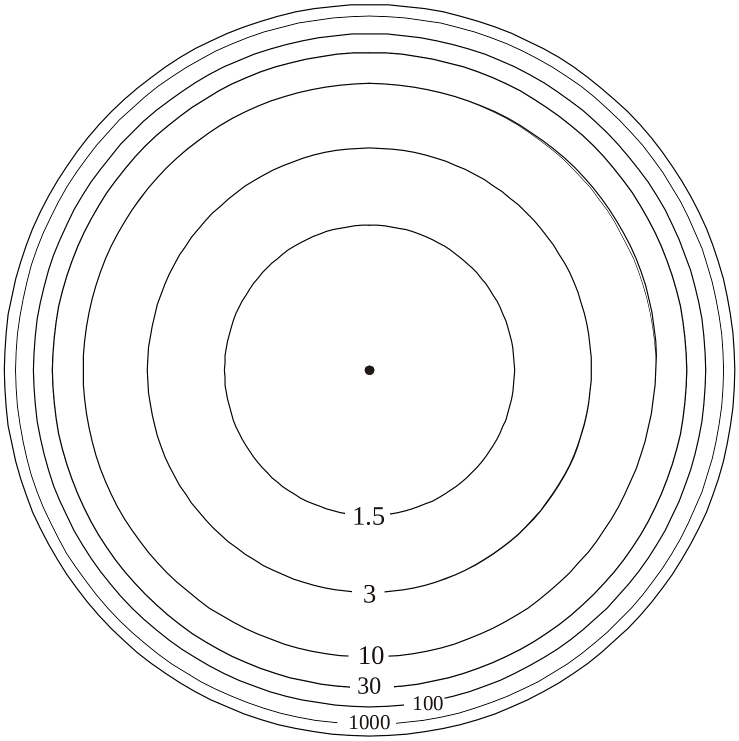

Figure 4: Apparent relative mass density of universe seen from earth; boundaries at density values of 1.5, 3, 10, 30, 100, 1000; minimum and maximum radii at 7.5 and 18.8 billion light years.

Constant Over-All Size of Universe

Figure 2 showed that the apparent receding velocity of a galaxy approaches the apparent speed of light at large true distances. However, Fig. 1 showed that the apparent speed of light becomes very small at large true distances. Consequently the actual value of the apparent velocity of a galaxy becomes very small at great distances, even though the galaxy is traveling at nearly the speed of light.

Figure 5 shows the actual value of the apparent velocity in km/sec for a galaxy, versus the true galaxy distance. The apparent galaxy velocity closely follows the Hubble law out to 5 billion light years, and reaches a maximum velocity of 150,000 km/sec at 10 billion light years, which is half the speed of light measured on earth (300,000 km/sec). This figure was obtained by combining Eqs 3 and 6. (In earlier literature, the author incorrectly stated that Figure 5 shows the "true" galaxy velocity; whereas it really shows the "apparent" galaxy velocity expressed in terms of an absolute measure of velocity.)

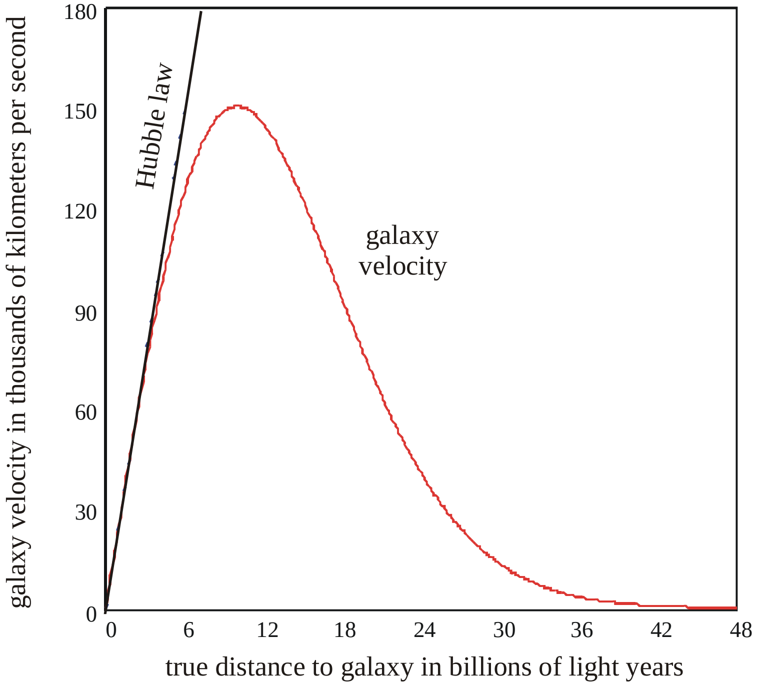

Figure 5: Apparent galaxy velocity expressed in thousands of kilometers per second compared with Hubble law.

At very great distances the actual value of the apparent velocity of a galaxy becomes very small. The universe expansion approaches zero over very great distances, and so the size of the universe remains constant, even though the universe expands about every point. How can the universe expand everywhere without getting bigger? A simple answer is that relativistic effects due to mass in the universe strongly compress dimensions at great distances. This relativistic compression offsets the expansion of the universe, and so the universe does not become any larger even though it expands about the location of every observer.

Uniqueness of Yilmaz Theory Predictions

When the Einstein gravitational field equation is applied, constraints that are somewhat arbitrary must be included to achieve a solution. Consequently multiple, contradictory solutions can be derived from the same physical model. In contrast, predictions from the Yilmaz gravitational theory are unique. Since a physical model can yield only one solution with the Yilmaz theory, the plots in Figs 1 to 5 are not arbitrary. They are uniquely constrained by (1) the formulas of the Yilmaz theory of gravity, (2) the simple cosmological postulate given earlier, and (3) specifying the average density of matter to achieve a Hubble constant of 20 km/sec per million light years.

The Steady-State Cosmology Theory

In 1948, Fred Hoyle and others proposed the ***Steady-State*** cosmology theory, which envisioned a universe that is infinitely old. The theory postulated that diffuse matter is continuously created to compensate for the expansion of the universe, and this diffuse matter forces the universe to expand. The rate of creation of matter needed to compensate for the universe expansion is far too small to be measured directly: only 2 hydrogen atoms created per year within a cubic kilometer. Although the Steady State theory became very popular, it was abandoned in the mid 1960's when Cosmic Microwave Background Radiation was discovered, because the Steady-State theory was unable to account for this strong cosmic microwave radiation being received from all directions of the universe.

As shown in Fig. 2, the Yilmaz cosmology model predicts that the universe must expand, yet the model assumes a constant density of matter. To satisfy both conditions requires that matter be continuously created to offset the universe expansion. Hence the Yilmaz cosmology model predicts a universe that is similar in many respects to Hoyle's Steady-State theory. However, unlike the Hoyle Steady-State theory, the Yilmaz cosmology model actually predicts Cosmic Microwave Background Radiation. The Hoyle Steady-State theory was a ***postulate***, which was analyzed using the Einstein gravitational field equation. The steady-state cosmology model derived from the Yilmaz gravitational theory is a ***prediction***, which was calculated ***rigorously*** from the Yilmaz theory.

Cosmic Microwave Background Radiation

The ***Cosmic Microwave Background Radiation*** could not be explained by the Hoyle steady-state theory without an artificial postulate. Nevertheless, it is directly predicted by the Yilmaz steady-state cosmology model. Figure 2 shows that at great distances the velocity of a galaxy gradually approaches the speed of light, and so galaxy spectra are strongly shifted to low frequencies. *Believe* [1], Appendix D gave an approximate analysis of this effect to show that light from very distant galaxies should produce cosmic microwave background radiation. This analysis is extended by this website in *Addendum* document *5,1 Calculation of Cosmic Background Radiation*, to yield the accurate results plotted in Fig. 6.

Because the light from a galaxy is Doppler shifted toward lower frequency, the equivalent blackbody temperature of the spectrum received from a galaxy decreases with distance to the galaxy. This Doppler shift was calculated by applying the expression for galaxy velocity in Eq. 6 to the Einstein formula for Doppler wavelength shift. The spectra of all stars are approximated by that of our sun, to give the plot shown in Fig. 7. The equivalent blackbody temperature decreases from 5770 °K (the blackbody temperature of the sun) for a close galaxy down to 1 °K for a galaxy at a true distance of 60 billion light-years.

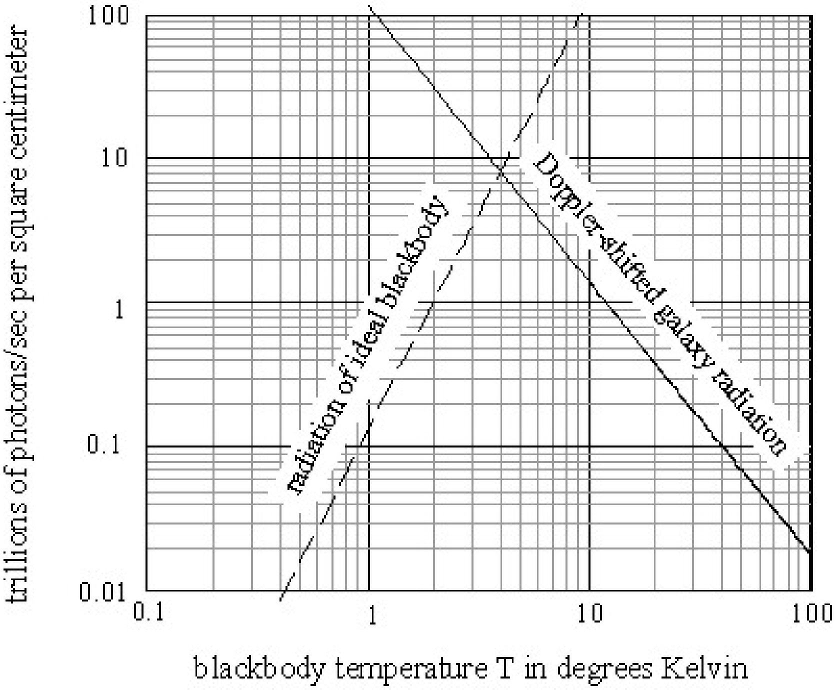

Figure 6: Photon rate intensity of cosmic radiation received from distant galaxies versus blackbody temperature of radiation; received radiation (solid), ideal radiation (dashed).

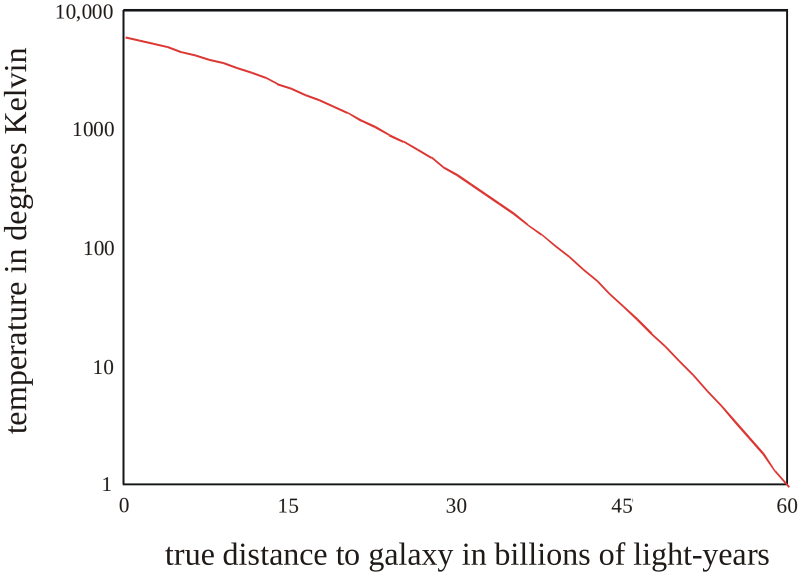

Figure 7: Blackbody temperature of radiation received from a distant galaxy versus the true distance r to the galaxy

The Hoyle Steady-State theory assumed that the universe expansion follows the Hubble Law, which rapidly passes through the region close to the speed of light that would generate microwave radiation, and so this theory cannot predict appreciable microwave radiation. In contrast, the Yilmaz steady-state cosmology model predicts very high radiation in the microwave region. The solid curve in Fig. 6 shows (for the Yilmaz cosmology model) the intensity of the received radiation versus the equivalent blackbody temperature of the spectrum. This curve gives the photon rate falling onto a square centimeter of receiver surface, as a function of the equivalent blackbody temperature T of the received Doppler-shifted spectrum. The blackbody temperature T is proportional to the average frequency of the received spectrum

An ideal blackbody emits a photon rate proportional to the cube of the blackbody temperature. The dashed plot in Fig 6 shows the photon rate per unit area that is emitted from an ideal blackbody. For an ideal blackbody, the radiation is in thermal equilibrium with the molecules at the surface. We assume that this radiation level cannot be exceeded by cosmic radiation in space. If it were, diffuse matter in space should rapidly absorb the cosmic radiation.

Hence the Doppler-shifted galaxy radiation shown by the solid plot in Fig 6 cannot exceed the dashed plot. This indicates that the intersection point of the two plots gives the effective blackbody temperature of the received blackbody radiation, which is 4.0 °K. Figure 7 shows that this radiation at 4.0 °K comes from stars at a distance of 56 billion light years.

The COBE satellite found that Cosmic Microwave Background Radiation is equivalent to radiation from a blackbody at a temperature of 2.73 °K. Our computed blackbody temperature (4.0 °K) differs from the COBE temperature by only 46 percent. The COBE radiation was received with very high uniformity from all directions, which agrees with our model. An important source of error in this analysis is that the spectra of all stars were approximated by the spectrum of our sun. For comparison, the blackbody temperature of the Cosmic Microwave Background Radiation predicted by the Big Bang theory varied from 5 to 30 °K.

Application of Yilmaz Model to Supernova Data

In recent years it was discovered that a particular type of supernova called Type 1a (which can be identified by its spectrum) radiates a nearly constant amount of peak power. Consequently type 1a supernovae can be used to measure astronomical distances. Since these supernovae emit a peak power equivalent to several billion suns, they can be observed in very distant galaxies.

The peak absolute magnitude of a Type 1a supernova varies from -18.9 to -19.9. Absolute magnitude is the equivalent magnitude of a star at a distance of 32.6 light years (10 parsecs). Since the absolute magnitude of the sun is 4.8, the peak power of a Type 1a supernova varies from 3.0 to 7.6 billion suns. As shown by Perlmutter [8] (p. 54, Fig. 1a), the shape of the supernova burst allows this variation to be compensated within ± 0.1 magnitude. This compensation is called "stretch correction".

Figure 9 shows the processed data from Type 1a supernovae given by Knop et. al [9], (page 117, Fig. 4). Curve (1) is the best-fit plot of the Knop data, and Curve (2) is the plot for the

Yilmaz cosmology model. To apply the Knop data, we need the peak absolute magnitude of the stretch-corrected supernova curve, which apparently is -19.4. This was not in the Knop article; it was determined by relating Fig. 9 to data in Perlmutter and Schmidt [10], (p. 117, Fig. 4).

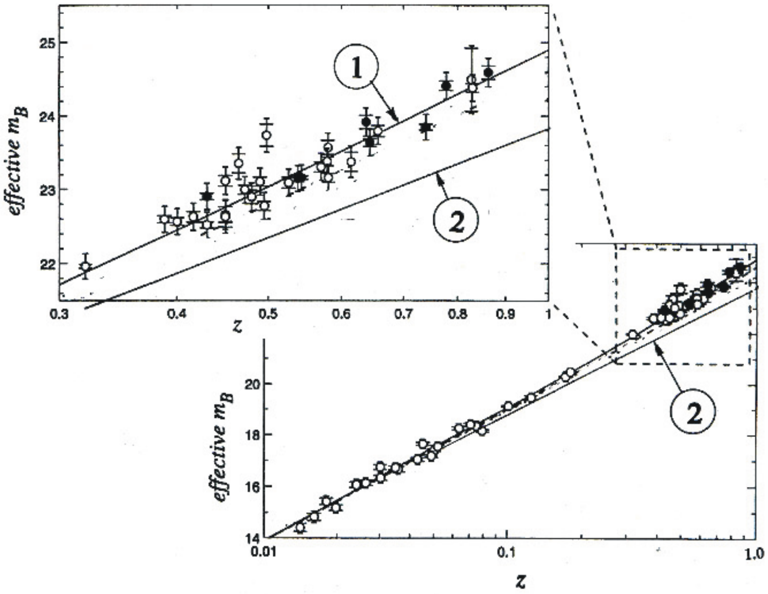

Figure 9: Type 1a supernova data applied to Yilmaz theory;
(1) average of data, (2) Yilmaz cosmology model

The horizontal axis shows the redshift (z), which represents (Δλ/λ), where (λ) is the normal wavelength of light observed here on earth and (Δλ) is the increase of wavelength of the light from the supernova. The vertical axis gives the effective magnitude of the observed light, corrected for supernova duration and various errors.

A magnitude difference in the light from two stars is (- 2.5) times the logarithm of the power ratio received from the stars, the higher the magnitude, the weaker is the received light. All of the supernovae in Fig 9 radiate nearly the same power. When two stars radiate the same power, the received light is inversely proportional to the square of their distances. The magnitude difference between these stars is ideally 5.0 times the logarithm of the distance ratio. A magnitude increase of 1.5 ideally represents a factor of 2 increase in distance, and a magnitude increase of 5 ideally represents a factor of 10 increase in distance.

Hubble Constant Derived from Supernova Data

The average curve (1) in Fig. 9 shows that at a redshift (z = Δλ/λ) of 0.01, the effective magnitude is 13.9. This exceeds the absolute peak magnitude of the stretch-corrected supernova (-19.4) by 33.3, which is equivalent to a distance ratio of 4.57 million. Absolute magnitude is defined in terms of a star at a distance of 10 parsecs (or 32.6 light years), and so the distance of the average curve at z = 0.01 is 4.57 million times 10 parsecs, which is 45.7 million parsecs, or 45.7 mega-parsecs (denoted 45.7 Mpc). The velocity ratio V/c is related to z by:

$$V/c = [(1 + z)^2 - 1]/[(1 + z)^2 + 1] \quad (7)$$

For z = .01, the velocity V is

$$V = 0.00995\ c = 2985\ m/sec \quad (8)$$

The calculated Hubble constant is

$$H_0 = (2985\ m/s)/(45.7\ Mpc) = 65.3\ m/sec\ per\ megaparsec \quad (9)$$

Since one parsec is 3.26 light years, this is equivalent to 20.03 km/sec per million light years. Our assumed Hubble constant is 20 km/sec per million light years.

The universe expansion predicted by the Yilmaz cosmology model is calculated from Eq. 6. At a redshift z of 1.00, Eq. 7 shows that V/c is 0.600, and Eq. 6 shows that distance (r) is 10.02 billion light years (Lyr). If power remained constant, the peak magnitude of a stretch-corrected supernova at this distance (r = 10.02 billion Lyr) would be

$$Magnitude = 5 \log[r/(32.6\ Lyr)] - 19.4 = 23.04 \quad (8)$$

If a supernova is moved to high redshift, its photon rate should remain constant, not its power. Energy per photon decreases by (1 + z), and so the received power should decrease by this factor. For a redshift z of 1, this is a factor of 2 decrease in power, or a magnitude increase of 0.75, which is 2.5 log(2). Adding 0.75 to Eq. 8 gives a magnitude of 23.8 for the Yilmaz cosmology model at redshift z = 1, which is the value of curve 2 in Fig. 9 at this redshift. The best-fit curve (1) magnitude is 24.9 at z = 1.0.

Relativistic Aspects of Stretch Correction

We can approximate stretch correction by considering the duration of a supernova burst 1.0 magnitude below the peak. Figure 1 in Perlmutter [8] (p. 54) shows that this duration varies from 19.3 days for -18.9 absolute peak magnitude to 26.7 days for -20.0 absolute peak magnitude. This represents a 1.1 magnitude decrease for a factor of 1.38 increased in duration, or 1.0 magnitude decrease for a 1.25 duration increase.

Figure 1 shows that, at a galaxy distance of 10 billion light years, a clock rate decreases to 80 percent. If a nearby supernova were moved 10 billion light years away, its duration would increase by 1.25 (which is 1/0.80) because of this clock rate decrease. This would cause a false 1.0 magnitude increase in stretch correction. Removing this error would reduce the effective magnitude at z = 1.0 of the best-fit curve (1) from 24.9 to 23.9. With this correction, curve (1) would nearly coincide with curve (2).

Thus, the Yilmaz cosmology model predicts radiation from Type 1a supernovae that closely agrees with measured data. Nevertheless, this model predicts a steady-state universe with no dark energy

Comments by Huseyin Yilmaz

In 2007, Prof. Huseyin Yilmaz gave the following comments concerning this cosmology model and my presentation of his gravitational theory:

"In my 1958 paper [7] describing a new theory of gravity, I applied the theory to a cosmology model. This was merely an example to illustrate the application of the new gravitational theory and was not intended to be a foundation for explaining the mysteries of cosmology.

"In the years since the passing of Albert Einstein in 1955, it has become apparent that the Einstein gravitational field equation does not provide a mathematically rigorous specification of the principles of relativity. For example, this equation has been used in computer studies to establish the physical reality of the black-hole singularity, even though a singularity severely violates physical evidence and Einstein absolutely rejected all singularity predictions derived from his theory.

"Before scientists can develop a meaningful explanation of cosmology, they must first achieve a rigorous relativistic theory of gravity. The simplified description of the new gravitational theory presented by Adrian Bjornson should be helpful in achieving that goal. His application of the new gravitational theory to cosmology illustrates the potential use of the new theory in studying cosmology. However, at this time I think it is premature for me either to endorse or reject the cosmological predictions that he has derived "

Appendix A
Application of Geodesic Equation to Yilmaz Cosmology Model

This appendix applies the geodesic equation to the Yilmaz model. By Tolman [11] (page 495, Eq. 20), the geodesic equation is:

$$d^2x^\alpha/ds^2 = - \Sigma_\mu \Sigma_\nu \Gamma_{\mu\nu}{}^\alpha u^\mu u^\nu \qquad \text{(A-1)}$$

where $\Gamma_{\mu\nu}{}^\alpha$ is the Christoffel symbol, and u^μ represents dx^μ/ds. This website shows in *Addendum* document *5,D Analysis of Geodesic Equations* (Eqs. 56 to 58) that this equation yields the following when the Yilmaz theory is applied to a ***static, spherically symmetric gravitational field***:

$$d\tau/ds = e^{2\phi} \qquad \text{(A-2)}$$

$$d^2r/ds^2 = (\partial\phi/\partial r)[1 - (dr/ds)^2] + [(\partial\phi/\partial r) + (1/r)](dx_t/ds)^2 \qquad \text{(A-3)}$$

$$d^2x_t/ds^2 = - [2(\partial\phi/\partial r) + (1/r)](dr/ds)(dx_t/ds) \qquad \text{(A-4)}$$

where dr represents radial motion, and dx_t represents tangential motion. The ***metric equation*** for the static Yilmaz theory in terms of radial and tangential motions is:

$$(ds)^2 = e^{-2\phi} (d\tau)^2 + e^{2\phi} [(dr)^2 + (dx_t)^2] \tag{A-5}$$

To apply these equations to the Yilmaz cosmology model, the gravitational potential 2ϕ is set equal to $(r/r_0)^2$. We can set the tangential motion dx_t equal to zero, and thereby limit our attention to radial motion. Geodesic equations A-2 to A-4 reduce to

$$d\tau/ds = \exp[(r/r_0)^2] \tag{A-6}$$

$$d^2r/ds^2 = (r/r_0{}^2) - (r/r_0{}^2)(dr/ds)^2 \tag{A-7}$$

The metric equation A-5 becomes

$$(ds)^2 = \exp[-(r/r_0)^2](d\tau)^2 - \exp[(r/r_0)^2](dr)^2 \tag{A-8}$$

Solve Eq. A-8 for $(dr)^2$ and divide by $(ds)^2$ to obtain:

$$(dr/ds)^2 = \exp[-2(r/r_0)^2](d\tau/ds)^2 - \exp[-(r/r_0)^2] \tag{A-9}$$

Square the expression for $d\tau/ds$ in Eq. A-6, and substitute it for $(d\tau/ds)^2$ in Eq. A-9. The term is unity and Eq. A-9 becomes

$$(dr/ds)^2 = 1 - \exp[-(r/r_0)^2] \tag{A-10}$$

Comparing Eq. A-6 with Eq. 3 in the body of this paper shows that $(d\tau/ds)$ is equal to (c/c_{ap}). Hence (dr/ds) can be expressed as

$$dr/ds = (dr/d\tau)(d\tau/ds) = (dr/d\tau)\,(c/c_{ap}) \tag{A-11}$$

Relativistic time τ is equal to ct, and so $dr/d\tau$ is equal to

$$(dr/d\tau) = (1/c)(dr/dt) = V_{ap}/c \tag{A-12}$$

Derivative dr/dt is the radial velocity of a galaxy as observed on earth. We call this the *apparent radial velocity*, which is denoted V_{ap}. Substituting Eq. A-12 into Eq. A-11 shows that dr/ds is equal to

$$dr/ds = V_{ap}/c_{ap} \tag{A-13}$$

Substituting this into Eq. A-10 gives our final formula

$$(V_{ap}/c_{ap})^2 = 1 - \exp[-(r/r_0)^2] \tag{A-14}$$

Appendix B
Calculating Metric Tensor for Yilmaz Cosmology Model

Derivation of Metric Tensor Formula

To calculate the metric tensor elements of the Yilmaz cosmology model, we must find the gravitational potential for this model, which postulates a constant average mass density throughout the universe. We apply the Poisson equation, which is

$$\nabla^2\phi' = -4\pi\rho \qquad \text{(B-1)}$$

Variable (ϕ') is the gravitational potential and (ρ) is the density of matter. The operator (∇^2) is called the Laplacian, which is defined by

$$\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2) \qquad \text{(B-2)}$$

The gravitational potential (ϕ') in Eq. B-1 is expressed in standard mass units, whereas the gravitational potential (ϕ) used by Yilmaz is expressed in normalized mass units. For a collection of masses having spherical symmetry, the gravitational potential (ϕ') at a given point (p), expressed in standard mass units, is

$$\phi' = M_1/r_1 + M_2/r_2 + M_3/r_3 + \text{etc.} \qquad \text{(B-3)}$$

where r_1, r_2, r_3 are the distances from the point (p) to the centers of masses M_1, M_2, M_3. In contrast, the gravitational potential (ϕ) used by Yilmaz, expressed in normalized mass units, is

$$\phi = m_1/r_1 + m_2/r_2 + m_3/r_3 + \text{etc.} \qquad \text{(B-4)}$$

The normalized mass (m) is calculated from the standard mass (M) by

$$m = (G/c^2)M \qquad \text{(B-5)}$$

Hence, the Yilmaz gravitational potential (ϕ) in normalized mass units is related as follows to the gravitational potential (ϕ') in standard mass units, which is used in Eq. B-1:

$$\phi = (G/c^2)\,\phi' \qquad \text{(B-6)}$$

Combining Eqs B-1 and B-6 gives the following modified Poisson equation with gravitational potential expressed in normalized mass units:

$$\nabla^2\phi = -4\pi(G/c^2)\rho \qquad \text{(B-7)}$$

Hildebrand [12] (p. 329) shows that the Laplacian of (ϕ) is expressed as follows in spherical coordinates:

$$\nabla^2\phi \;=\; (1/r^2)\partial_r\{r^2\partial_r\phi\} + (1/r^2\sin\theta)\partial_\theta\{\sin\theta\;\partial_\theta\phi\} + (1/r^2\sin^2\theta)\partial_\psi{}^2\phi \qquad \text{(B-8)}$$

The Yilmaz cosmology model assumes that the gravitational field is spherically symmetric. With spherical symmetry, the derivatives of the gravitational potential (ϕ) relative to angles θ and ψ (denoted by ∂_θ, ∂_ψ) are zero. Hence Eq. B-8 reduces to

$$\nabla^2\phi \;=\; (1/r^2)\partial_r\{r^2\partial_r\phi\} \qquad \textit{(spherical symmetry)} \qquad \text{(B-9)}$$

Applying this to the modified Poisson's equation of Eq. B-7 gives

$$\nabla^2\phi \;=\; (1/r^2)\;\partial\{r^2\partial\phi/\partial r\}/\partial r = -4\pi(G/c^2)\rho \qquad \text{(B-10)}$$

The quantity $\partial_r\phi$ was replaced by $\partial\phi/\partial r$. Since we are considering only a single variable r, the partial derivatives can be replaced by simple derivatives, and Eq. B-10 becomes

$$d\{r^2 d\phi/dr\} = -(4\pi G/c^2)\;\rho r^2\;dr \qquad \text{(B-11)}$$

The Yilmaz cosmology model postulates that the mass density (ρ) is constant, and so the integral of this is

$$r^2 d\phi/dr \;=\; -(4\pi G\rho/c^2) \int r^2\,dr \;=\; -(4\pi G\rho/c^2)\,(r^3/3) + C_1 \qquad \text{(B-12)}$$

where C_1 is a constant. Multiplying by (dr/r^2) gives

$$d\phi \;=\; -(4\pi G\rho/3c^2)\,(\,r\,dr) \;+\; C_1\,(dr/r^2) \qquad \text{(B-13)}$$

Integrating this gives

$$\phi \;=\; -(2\pi G\rho/3c^2)\,r^2 \;+\; C_2 - \;C_1/r \qquad \text{(B-14)}$$

where C_2 is a constant. The constant C_1 must be zero; otherwise the gravitational potential would be infinite at ($r = 0$). Gravitational potential can be arbitrarily defined as zero at any point, because it is the difference in gravitational potential that is important. The gravitational potential is defined to be zero at the point ($r = 0$), and so Eq. B-14 becomes

$$\phi \;=\; -(2\pi G\rho/3c^2)\,r^2 \qquad \text{(B-15)}$$

This gives the gravitational potential on earth relative to the distant galaxy. To obtain the gravitational potential of the distant galaxy relative to earth, we take the negative of Eq. B-15. In the equations for the Yilmaz metric tensor elements we need the quantity (2ϕ), which is

$$2\phi \;=\; (4\pi G\rho/3c^2)\,r^2 \qquad \text{(B-16)}$$

To simplify our calculations, it is convenient to define the parameter (r_0) by

$$r_0^2 = 3c^2/4\pi\rho G \qquad \text{(B-17)}$$

Applying Eq. B-17 to Eq. B-16 gives

$$2\phi = (r/r_0)^2 \qquad \text{(B-18)}$$

This website gives in Eq. 16 of *1,3 Yilmaz Relativistic Theory of Gravity* the following formulas for the elements of the metric tensor expressed in terms of the gravitational potential (ϕ):

$$g_{00} = \exp(-2\phi) \ , \quad g_{11} = g_{22} = g_{33} = -\exp(2\phi) \qquad \text{(B-19)}$$

Substituting Eq. B-18 into Eq. B-19 gives the following metric tensor elements for the Yilmaz cosmology model:

$$g_{00} = \exp[-(r/r_0)^2] \ , \quad g_{11} = g_{22} = g_{33} = -\exp[(r/r_0)^2] \qquad \text{(B-20)}$$

When Yilmaz presented his cosmology model in the first paper on his theory, he specified the parameter (r_0) by the following formula, instead of that in Eq. B-17:

$$r_0^2 = 3c^2/8\pi\rho G \qquad \textit{(Statistical mass distribution)} \qquad \text{(B-21)}$$

The Yilmaz analysis modeled the universe as a uniform statistical distribution of point masses, which the author initially assumed was more realistic than the assumption of uniform mass density. This assumption would be better if the mass of the universe were contained primarily in the stars. However, there is much more mass in the highly diffuse matter contained in the enormous space between the stars than there is within the stars themselves. Consequently, the simpler assumption of uniform mass distribution specified by Eq. B-17 is much more realistic.

Predicted Mass Density of Universe

Equation B-17 gives the following formula relating the parameter (r_0) to the average mass density (ρ) of the universe:

$$r_0^2 = 3c^2/4\pi\rho G \qquad \text{(B-22)}$$

Figure 2 showed that this parameter (r_0) is the apparent radius of the “observable universe” that is derived from the Hubble Law. Recent astronomical data indicates a Hubble constant of 20 km/sec per million light years of galaxy distance, and this corresponds to an (r_0) of 15 billion light years. Solving Eq. B-22 for the density (ρ) gives

$$\rho = 3c^2/4\pi G r_0^2 = 3/4\pi G T_0^2 \qquad \text{(B-23)}$$

Parameter (T_0) is equal to (r_0/c), which is the apparent age of the universe. For (r_0) equal to 15 billion light years, (T_0) is 15 billion years. The parameters in Eq. B-23 are equal to

$$T_0 = 15\text{x}10^9 \text{ year} = (15\text{x}10^9 \text{ yr})(31.56\text{x}10^6 \text{ sec/yr}) = 4.734\text{x}10^{17} \text{ sec} \qquad \text{(B-24)}$$

$$G = 6.674\text{x}10^{-14}\ \text{m}^3/\text{gm-sec}^2 \qquad \text{(B-25)}$$

Substituting Eqs. B-24, B-25 into Eq. B-23 gives

$$\rho = 15.96\text{x}10^{-24}\ \text{gm/m}^3 \qquad \text{(B-26)}$$

The mass of one hydrogen atom is

$$\text{H-atom mass} = 1.67\text{x}10^{-24}\ \text{gm} \qquad \text{(B-27)}$$

Comparing Eqs B-26, B-27 shows that the predicted average mass density (ρ) of the universe is equivalent to 9.56 hydrogen atoms per cubic meter.

Reference [13] shows in its “Table” that the measured “average mass density” of the universe derived from observations is (5 to 12)x10^{-30} gram/cm^3, or (5 to 12)x10^{-24} gram/meter^3. Comparing this with the hydrogen-atom mass in Eq. B-27 shows that this measured value of average mass density of the universe is equivalent to (3.0 to 7.2) hydrogen atoms per cubic meter. In comparison, the Yilmaz cosmology model predicts an average mass density of 9.6 hydrogen atoms per cubic meter, which is only 33% above this measured range.

References

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