5,4 Addendum Chapter 4
Application of Geodesic Equations to the Yilmaz Theory

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1, Use of the Geodesic Equations

The Newtonian concept of a gravitational force is not included in general relativity. Its effect is replaced by the *curvature of space*. The curvature of space is characterized by a set of differential equations that are called the *geodesic equations*.

When the orbit of a planet is calculated using Newtonian theory, one applies Newton's law of gravitational attraction and Newton's law of motion, which states that force is equal to mass times acceleration. We can combine these into a single equation, which states that the acceleration of a body is equal to the gravitational force applied to the body divided by its mass. Acceleration is the derivative of velocity, and so is the second derivative of position. Thus, with Newtonian theory, the second derivative of the position of a body is equal to the gravitational force applied to the body divided by the mass of the body.

The calculations are entirely different with relativity, because this theory does not allow the concept of a gravitational force. With general relativity, the orbit of a planet is calculated from a set of four equations, which are called the *geodesic equations*. These four equations specify the second derivatives, relative to the line element ds along the planetary path, of the four space-time coordinate variables. In rectangular coordinates, these four space-time coordinates are τ, x, y, z. In the weak gravitational fields of our solar system, an increment Δs along a space-time planetary trajectory is closely approximated by $\Delta\tau$.

In (5,D) *Addendum*, Appendix D general geodesic equations in spherical coordinates are calculated for a static, spherically symmetric gravitational field. From these, specific geodesic equations are derived for two applications of the Yilmaz theory: (a) the single-star model and (b) the Yilmaz cosmology model. These specific geodesic equations are given in Section 2. From the geodesic equations of the Yilmaz cosmology model, Section 3 calculates the radial velocity of a galaxy.

General geodesic equations for the static Yilmaz theory, expressed in rectangular coordinates, are also derived in (5,D) *Addendum*, Appendix D. These are simplified to give practical formulas for calculating the orbits of planets and other bodies in our solar system. These equations are given in Section 4.

2, Yilmaz Geodesic Equations for a Static, Spherically Symmetric Gravitational Field

The geodesic equations of the Yilmaz theory for a static, spherically symmetric gravitational field are derived in (5,D) *Addendum*, Appendix D.. These are expressed in terms of motions in the radial and tangential directions. The general geodesic equations for the static Yilmaz theory are derived in Eqs. D-56 to D-58, and are as follows:

General Static, Spherically Symmetric Gravitational Field

$$d\tau/ds = e^{2\phi} \qquad (1)$$

$$d^2r/ds^2 = (\partial\phi/\partial r)\{ 1 - (dr/ds)^2 \} + \{ (\partial\phi/\partial r) + (1/r)](dx_t/ds)^2 \qquad (2)$$

$$d^2x_t/ds^2 = - \{ 2(\partial\phi/\partial r) + 1/r \} (dr/ds)(dx_t/ds) \qquad (3)$$

The variable dr represents incremental motion in the radial direction, and dx_t represents incremental motion in the tangential direction, which is perpendicular to the radius.

These general formulas are applied to the Yilmaz theory solutions for a single star and to the Yilmaz cosmology model. In the Yilmaz theory solution for a single star, the gravitational potential ϕ is equal to m/r. The resultant geodesic equations for the Yilmaz single-star model are given in Eqs. D-61 to D-63 of (5,D) *Addendum*, Appendix D, and are as follows:

Yilmaz Single Star Model

$$d\tau/ds = e^{2m/r} \qquad (4)$$

$$d^2r/ds^2 = (1/r)(m/r)\{ (dr/ds)^2 - 1\} + (1/r)[1 - (m/r)\} (dx_t/ds)^2 \qquad (5)$$

$$d^2x_t/ds^2 = (1/r)\{ (2m/r) - 1 \} (dr/ds)(dx_t/ds) \qquad (6)$$

In the Yilmaz cosmology model, the gravitational potential variable 2ϕ is equal to $(r/r_0)^2$. The resultant geodesic equations for the Yilmaz cosmology model are given in Eqs. D-64 to D-66 of (5,D) *Addendum*, Appendix D, and are as follows:

Yilmaz Cosmology Model

$$d\tau/ds = \exp[(r/r_0)^2] \qquad (7)$$

$$d^2r/ds^2 = (r/r_0{}^2)\{ 1 - (dr/ds)^2 \} + (1/r)[1 + (r/r_0)^2](dx_t/ds)^2 \qquad (8)$$

$$d^2x_t/ds^2 = - (1/r)[1 + 2(r/r_0)^2](dr/ds)(dx_t/ds) \qquad (9)$$

When these geodesic equations are applied, one often needs the metric equation. The general metric equation for the static Yilmaz theory is expressed as follows in terms of motions in the radial and tangential directions:

$$(ds)^2 = e^{-2\phi} (d\tau)^2 + e^{2\phi} [(dr)^2 + (dx_t)^2] \qquad (10)$$

As shown in Eqs. D-107, D-108 of (5,D) *Addendum* Appendix D, the geodesic equations for the Yilmaz Single-Star Model given in Eqs. 4 to 6 can be expressed directly in terms of normalized time as the following two equations:

$$r(d^2r/d\tau^2) = 3(m/r)(dr/d\tau)^2 - (m/r)e^{-4m/r} + [1 - (m/r)](dx_t/d\tau)^2 \qquad (11)$$

$$r(d^2x_t/d\tau^2) = \{(4m/r) - 1\}(dr/d\tau)(dx_t/d\tau) \qquad (12)$$

Equations 4-11, 4-12 can be expressed as

$$rA_r = 3(m/r)V_r^2 - (m/r)c^2e^{-4m/r} + [1 - (m/r)]V_t^2 \qquad (13)$$

$$rA_t = \{(4m/r) - 1\}V_r V_t \qquad (14)$$

where V_r and V_t are the radial and tangential velocities dr/dt and dx_t/dt; and A_r and A_t are the radial and tangential accelerations dr^2/dt^2 and dx_t^2/dt^2.

3, Solution of Yilmaz Cosmology Model for Radial Motion

This section derives the formula that specifies the radial velocity of a galaxy for the Yilmaz cosmology model. We can simplify the equations for the Yilmaz cosmology model by setting tangential incremental motion dx_t equal to zero, and thereby limiting our attention to radial motion. The metric equation formula of Eq. 10 becomes

$$(ds)^2 = e^{-2\phi}(d\tau)^2 + e^{2\phi}(dr)^2 = \exp[-(r/r_0)^2](d\tau)^2 - \exp[(r/r_0)^2](dr)^2 \qquad (15)$$

where 2ϕ was set equal to $(r/r_0)^2$. When dx_t in Eqs. 7 to 9 is set to zero, the geodesic equations reduce to

$$d\tau/ds = \exp[(r/r_0)^2] \qquad (16)$$

$$d^2r/ds^2 = (r/r_0^2) - (r/r_0^2)(dr/ds)^2 \qquad (17)$$

Solve the metric equation of Eq. 15 for $(dr)^2$:

$$(dr)^2 = \exp[-2(r/r_0)^2](d\tau)^2 - \exp[-(r/r_0)^2](ds)^2 \qquad (18)$$

Divide Eq. 18 by $(ds)^2$ to obtain the ratio $(dr/ds)^2$:

$$(dr/ds)^2 = \exp[-2(r/r_0)^2](d\tau/ds)^2 - \exp[-(r/r_0)^2] \qquad (19)$$

Square the expression for $d\tau/ds$ in Eq. 16, and substitute it for $(d\tau/ds)^2$ in Eq. 19. The first term is unity and so Eq. 19 becomes

$$(dr/ds)^2 = 1 - \exp[-(r/r_0)^2] \qquad (20)$$

Substituting Eq. 20 into Eq. 17 gives

$$d^2r/ds^2 = (r/r_o^2) \exp[-(r/r_0)^2] \qquad (21)$$

Differentiating Eq. 4-20 relative to s gives

$$2(dr/ds)(d^2r/ds^2) = - \exp[-(r/r_0)^2]\{ (-2)(r/r_0^2) (dr/ds) \} \qquad (22)$$

Dividing this by 2(dr/ds) gives the same formula as Eq. 21. Hence we have proven that the equations are consistent. The derivative dr/ds can be expressed as follows by applying the expression for $d\tau/ds$ in Eq. 16:

$$dr/ds = (dr/d\tau)(d\tau/ds) = (dr/d\tau) \exp[(r/r_0)^2] \qquad (23)$$

In Appendix C of *Believe* [1], Table C-2 shows that the apparent relative speed of light c_{ap}/c is equal to

$$c_{ap}/c = \exp[-(r/r_0)^2] \qquad (24)$$

Combining Eqs. 23, 24 gives

$$dr/ds = (dr/d\tau) (c/c_{ap}) \qquad (25)$$

Since normalized time τ is equal to ct, the derivative $dr/d\tau$ can be expressed as follows in terms of real time t:

$$(dr/d\tau) = (1/c)(dr/dt) = (1/c)V_{ap} \qquad (26)$$

The derivative dr/dt is the velocity of a distant galaxy as observed on earth. We call this the *apparent velocity*, which is denoted V_{ap}. Substituting Eq. 26 into Eq. 25 shows that dr/ds is equal to

$$dr/ds = V_{ap}/c_{ap} \qquad (27)$$

Substituting this into Eq. 4-20 gives our final formula

$$(V_{ap}/c_{ap})^2 = 1 - \exp[-(r/r_0)^2] \qquad (28)$$

This gives the apparent radial velocity V_{ap} of a galaxy, divided by the apparent speed of light c_{ap}, expressed in terms of the true distance r to the galaxy.

4, Practical Geodesic Equations for Multi-Body Computations Using the Yilmaz Theory

In Eqs. D-86, D-87 of (5,D) *Addendum*, Appendix D the following geodesic equations for the general static Yilmaz theory expressed in rectangular coordinates are derived:

$$d^2\tau/ds^2 = 2\,(d\tau/ds)\ \Sigma_k\ \partial_k\phi\ (dx^k/ds) \tag{29}$$

$$d^2x^j/ds^2 = e^{-4\phi}\ \partial_j\phi\ (d\tau/ds)^2 - \partial_j\phi\ \Sigma_{k(k\neq j)}\ (dx^k/ds)^2 - 2(dx^j/ds)\ \Sigma_k\ \partial_k\phi\ (dx^k/ds) \tag{30}$$

The indices j, k have the values 1, 2, 3. The summation in the last term sums over the three values (1, 2, 3) of the index k, but the summation in the middle term omits the value for k that is equal to j.

Within our solar system, these geodesic equations can be greatly simplified by using an approximation that holds to very high accuracy. Equations 1, 4 showed that the derivative $d\tau/ds$ is as follows in the Yilmaz-theory solution for a single star:

$$d\tau/ds = e^{2\phi} = e^{2m/r} \tag{31}$$

Within our solar system, the maximum value of 2m/r is 4.2 x 10^{-6}, and so $d\tau/ds$ is closely approximated by [1 + 2(m/r)], which is very close to unity. This shows that the gravitational field of the sun has a very small effect on the $d\tau/ds$ ratio, and so the effect on the $d\tau/ds$ ratio of the gravitational fields of the planets is infinitesimal. This argument allows us to approximate $d\tau/ds$ by $e^{2\phi}$ in the geodesic equations and still achieve a very accurate solution. With this approach, (5,D) *Addendum*, Appendix D derives in Eqs D-93 to D-96 the following approximate geodesic equations, which hold to very high accuracy in our solar system:

$$A_x = (\partial\phi/\partial x)(c^2e^{-4\phi} - V^2) - 2(\partial\phi/\partial y)V_xV_y - 2(\partial\phi/\partial z)V_xV_z \tag{32}$$

$$A_y = (\partial\phi/\partial y)(c^2e^{-4\phi} - V^2) - 2(\partial\phi/\partial x)V_yV_x - 2(\partial\phi/\partial z)V_yV_z \tag{33}$$

$$A_z = (\partial\phi/\partial z)(c^2e^{-4\phi} - V^2) - 2(\partial\phi/\partial x)V_zV_x - 2(\partial\phi/\partial y)V_zV_y \tag{34}$$

The variable V is the absolute value of velocity, given by

$$V^2 = V_x{}^2 + V_y{}^2 + V_z{}^2 \tag{35}$$

The variables A_x, A_y, A_z are the accelerations in the x, y, z directions, and V_x, V_y, V_z are the corresponding velocities. Appendix E of *Believe* [1] explains how these formulas can be applied to calculate with high accuracy the orbits of multiple bodies of the solar system.

References

[1] Adrian Bjornson, *A Universe that We Can Believe, Addison :Press, 2000, described in world-wide website Olduniverse.com* ISBN 09703231-0-7.