5,5 Addendum Chapter 5
General Time-Varying Yilmaz Theory

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This chapter analyzes the general time-varying Yilmaz theory. The static solution of the Yilmaz theory is derived by this website in 1,2 *Yilmaz Relativistic Theory of Gravity*. The static solution theoretically applies only when the gravitational potential ϕ does not vary with time, yet it gives a very accurate approximation if the velocities are small relative to the speed of light. When the gravitational potential ϕ varies with time, the full gravitational potential tensor $\phi_\mu{}^\nu$ must be considered to achieve an exact solution. The trace of the tensor $\phi_\mu{}^\nu$ (the sum of its diagonal elements) is the gravitational potential ϕ.

When an index in a tensor equation is repeated, with one index a subscript and the other a superscript, the corresponding term is to be summed over the four values (0, 1, 2, 3) of the index. This website normally indicates the implied summations with Σ summation signs, but this chapter omits the implied summations in some of the equations, in order to simplify the formulas. Comments are added to indicate that summation over specific indices is assumed.

1, Basic Equations for General Yilmaz Theory

The general time-varying Yilmaz theory is characterized by two basic formulas. One formula specifies the gravitational potential tensor $\phi_\mu{}^\nu$ in terms of the elements of mass and their velocity components. The second formula is a differential relation that specifies the metric tensor $g_{\mu\nu}$ in terms of the gravitational potential tensor $\phi_\mu{}^\nu$.

Formula for gravitational potential tensor. Yilmaz [5] (eq. 1.2) has shown that, in the general time-varying form of the Yilmaz theory, the ***gravitational potential tensor*** $\phi_\mu{}^\nu$ is calculated from the following integral:

$$\phi_\mu{}^\nu \ = \ f \ \{ \ u_\mu \, u^\nu)(dm/r) \ \}_{\text{retarded}} \qquad (1)$$

where dm is an element of mass and r is the absolute value of the distance from that mass element to the point at which the tensor is calculated. Yilmaz expressed (dm) as the product (ρ dv) where ρ is the mass density of the medium, and dv is an element of volume. The subscript "retarded" indicates that a time delay equal to r/c is included in the calculation between a change at the mass element (dm) and the resultant effect at the point (p) where the gravitational potential is calculated. This time delay is the time for a light pulse to propagate from the mass element to the point (p).

The variables u_μ, u^ν are the relativistic velocities of the mass element. The relativistic velocity u^ν is equal to dx^ν/ds. The relativistic velocity u_μ is equal to $g_{\mu\alpha}\, u^\alpha$. Hence u_μ is equal to $g_{\mu\alpha}\, (dx^\alpha/ds)$, which should be summed over the four values of the α index.

Formula for metric tensor. Yilmaz [5] (eq. 1.1) shows that the ***metric tensor*** $g_{\mu\nu}$ for the general time-varying Yilmaz theory is calculated from the gravitational potential tensor $\phi_\mu{}^\nu$ using the following differential formula:

$$dg_{\mu\nu} = 2g_{\mu\nu}\, d\phi - 2\, \Sigma_\alpha\{g_{\mu\alpha}\, d\phi_\nu{}^\alpha + g_{\alpha\nu}\, d\phi_\mu{}^\alpha\} \tag{2}$$

The variable ϕ is the trace of the gravitational potential $\phi_\mu{}^\nu$, which is the sum of its diagonal elements, as shown by

$$\phi = \phi_0{}^0 + \phi_1{}^1 + \phi_2{}^2 + \phi_3{}^3 \tag{3}$$

Document *5,F Addendum Appendix F* shows in Eqs. 3-4, 3-5 that Eq. 2 can also be expressed as

$$dg_{\mu\nu} = 2g_{\mu\nu}\, d\phi - 4\, \Sigma_\alpha\, g_{\mu\alpha}\, d\phi_\nu{}^\alpha \tag{4}$$

$$dg_{\mu\nu} = 2g_{\mu\nu}\, d\phi - 4\, \Sigma_\alpha\, g_{\alpha\nu}\, d\phi_\mu{}^\alpha \tag{5}$$

From the covariant metric tensor $g_{\mu\nu}$ one can calculate the contravariant metric tensor $g^{\mu\nu}$ by recognizing that the product of the matrices of the two tensors $g_{\mu\nu}$, $g^{\mu\nu}$ is a unit matrix.

2, Elements of the Yilmaz Gravitational Field Equation

Like the Einstein theory, the Yilmaz theory is based on its gravitational field equation. However, in the Yilmaz theory the gravitational field equation is not solved when the theory is applied, because the Yilmaz theory also yields an exact formula for the metric tensor. This chapter (supplemented by *5,F Appendix F*) will prove that the gravitational field equation is exactly satisfied by the metric tensor for the general time-varying Yilmaz theory.

The gravitational field equation for the Yilmaz theory is

$$G_\mu{}^\nu = R_\mu{}^\nu - \tfrac{1}{2}\, \delta_\mu{}^\nu\, R = -2(\tau_\mu{}^\nu + t_\mu{}^\nu) \tag{6}$$

The Yilmaz theory calls $\tau_\mu{}^\nu$ the "stress-energy tensor for matter", and $t_\mu{}^\nu$ is called the "stress-energy tensor for the gravitational field".

Formula for Ricci tensor. From the metric tensors $g_{\mu\nu}$, $g^{\mu\nu}$ computed from Eq. 2, one can calculate the Ricci tensor $R_\mu{}^\nu$ with the same formulas used with the Einstein theory. One first calculates the 64 Christoffel symbols from the following formula, given by Tolman (1934) (p. 494, eq. 18):

$$\Gamma_{\mu\nu}^{\alpha} = \tfrac{1}{2}\, g^{\alpha\beta}\{\partial_\nu g_{\mu\beta} + \partial_\mu g_{\nu\beta} - \partial_\beta g_{\mu\nu}\} \tag{7}$$

The symbol ∂_μ denotes the partial derivative relative to x^μ:

$$\partial_\mu \phi = \partial\phi/\partial x^\mu \tag{8}$$

Tolman [8] (p. 495, eq. 25) shows that the Ricci tensor is calculated from the Christoffel symbols by:

$$R_{\mu\nu} = \Gamma_{\mu\sigma}^{\alpha}\Gamma_{\alpha\nu}^{\sigma} - \Gamma_{\mu\nu}^{\alpha}\Gamma_{\alpha\sigma}^{\sigma} + \partial_{\nu}\Gamma_{\mu\sigma}^{\sigma} - \partial_{\sigma}\Gamma_{\mu\nu}^{\sigma} \qquad (9)$$

In accordance with Tolman [8] (p. 495, eq. 21) the mixed form of the Ricci tensor R_{μ}^{ν} is calculated from the covariant form $R_{\mu\nu}$ by:

$$R_{\mu}^{\nu} = g^{\nu\alpha} R_{\mu\alpha} \qquad (10)$$

In Eqs. 7 to 10, the terms with the repeated indices α, β, σ should be summed over the four values of the repeated indices.

In his documents, Yilmaz uses the negative of Eq. 9 to specify the Ricci tensor. This choice reverses the sign of the Ricci tensor R_{μ}^{ν} and thereby eliminates the negative sign in the right hand side of the gravitational field equation in Eq. 6. This website uses the Tolman definition for the Ricci tensor in Eq. 9, and includes a negative sign in the Yilmaz gravitational field equation.

Formulas for stress-energy tensors. Based on the formulas for the general Yilmaz theory given in Eqs. 1, 2, Addendum document 5,F Appendix F derives in Eqs. 1-34, 1-35 the following formulas for the stress-energy tensors for the general time-varying Yilmaz theory:

$$t_{\mu}^{\nu} = -2\partial_{\mu}\phi_{\beta}^{\alpha}\,\partial^{\nu}\phi_{\alpha}^{\beta} + \partial_{\mu}\phi\,\partial^{\nu}\phi + \delta_{\mu}^{\nu}\{\partial_{\lambda}\phi_{\beta}^{\alpha}\partial^{\lambda}\phi_{\alpha}^{\beta} - \tfrac{1}{2}\,\partial_{\lambda}\phi\partial^{\lambda}\phi\} \qquad (11)$$

$$\tau_{\mu}^{\nu} = \partial_{\alpha}\{\partial^{\alpha}\phi_{\mu}^{\nu} - \partial^{\nu}\phi_{\mu}^{\alpha}\} + 2\,\partial_{\alpha}\phi\{\partial^{\alpha}\phi_{\mu}^{\nu} - \partial^{\nu}\phi_{\mu}^{\alpha}\} \qquad (12)$$

In Eqs. 11, 12, the terms with the repeated indices α, β, λ should be summed over the four values of the repeated indices.

Addendum document *5,F Appendix F* proves that when these stress-energy formulas are applied to the gravitational field equation of Eq. 6, that the Yilmaz gravitational field equation is exactly satisfied. This shows that the Yilmaz gravitational field equation is automatically satisfied for the general time-varying Yilmaz theory, and so never has to be solved when the Yilmaz theory is applied. The general Yilmaz theory is implemented by solving the formulas for ϕ_{μ}^{ν} and $g_{\mu\nu}$ given in Eqs. 1, 2.

3, Determinant g of the Metric Tensor

The determinant of the covariant metric tensor $g_{\mu\nu}$ is denoted g. Section 3.3 of 5,F Appendix F proves that the determinant of $g_{\mu\nu}$ for the general time-varying Yilmaz theory has the following very simple formula:

$$g = \det|g_{\mu\nu}| = -e^{4\phi} \qquad (13)$$

The expression $\sqrt{[-g]}$, which is important in tensor analysis, is equal to

$$\sqrt{[-g]} = e^{2\phi} \qquad (14)$$

This formula for g greatly simplifies the calculation of the contravariant metric tensor $g^{\mu\nu}$ for general applications where the metric tensor is not diagonal. The product of the matrices of the metric tensors $g_{\mu\nu}$, $g^{\mu\nu}$ is a unit matrix. Hence it can be shown that the elements of the contravariant metric tensor are computed from

$$g^{\mu\nu} = \mathrm{cof}[g_{\mu\nu}]/\det|g_{\mu\nu}| = \mathrm{cof}[g_{\mu\nu}]/g \qquad (15)$$

The expression $\det|g_{\mu\nu}|$ is the determinant of the $g_{\mu\nu}$ matrix, which is the variable g. The expression $\mathrm{cof}[g_{\mu\nu}]$ is the cofactor of the element $g_{\mu\nu}$, which can be expressed as

$$\mathrm{cof}[g_{\mu\nu}] = (-1)^{\mu+1+\nu+1}\,\mathrm{minor}[g_{\mu\nu}] = (-1)^{\mu+\nu}\,\mathrm{minor}[g_{\mu\nu}] \qquad (16)$$

The expression $\mathrm{minor}[g_{\mu\nu}]$ denotes the minor of $g_{\mu\nu}$, which is obtained by deleting from the $g_{\mu\nu}$ matrix the row and column of the particular $g_{\mu\nu}$ element, and taking the determinant of the result. Combining Eqs. 13, 15, 16 gives the following formula for the elements of the contravariant metric tensor for the general Yilmaz theory:

$$g^{\mu\nu} = -(-1)^{\mu+\nu}\,e^{-4\phi}\,\mathrm{minor}[g_{\mu\nu}] \qquad (17)$$

When the covariant and contravariant metric tensors are known, one can readily convert any other tensor from one of its forms to another by means of the following formulas:

$$A^{\mu\nu} = \Sigma_\alpha\ g^{\mu\alpha} A_\alpha{}^\nu \qquad (18)$$

$$A_\mu{}^\nu = \Sigma_\alpha\ g^{\nu\alpha} A_{\mu\alpha} \qquad (19)$$

$$A_{\mu\nu} = \Sigma_\alpha\ g_{\nu\alpha} A_\mu{}^\alpha \qquad (20)$$

$$A_\mu{}^\nu = \Sigma_\alpha\ g_{\mu\alpha} A^{\alpha\nu} \qquad (21)$$

The factor $\sqrt{[-g]}$ is used to form the tensor density, which is a powerful concept in tensor analysis. A tensor density is traditionally represented by Old German script, but we use bold letters instead. A tensor density is formed by multiplying a tensor by $\sqrt{[-g]}$. The tensor density corresponding to the tensor $A_\mu{}^\nu$ is denoted $\mathbf{A}_\mu{}^\nu$ and is defined by:

$$\mathbf{A}_\mu{}^\nu = \sqrt{[-g]}A_\mu{}^\nu \qquad \textit{(tensor density)} \qquad (22)$$

In the Yilmaz theory, $\sqrt{[-g]}$ is always equal to ($e^{2\phi}$), even in the general time-varying theory, and so Eq. 22 becomes:

$$\mathbf{A}_\mu{}^\nu = e^{2\phi} A_\mu{}^\nu \qquad \textit{(tensor density for Yilmaz theory)} \qquad (23)$$

4, Conservation of Energy and Momentum

Landau and Lifshitz [7] (p. 280) explain that the conservation of energy and momentum for matter (including electromagnetic energy) requires that the following integral should be conserved: $f \sqrt{[-g}\ T_\mu{}^\nu\ dS_\nu$, where $\sqrt{[-g]}\ T_\mu{}^\nu$ is the tensor density for the energy-momentum tensor $T_\mu{}^\nu$. They state that to achieve this requires that the following condition must be satisfied:

$$\partial_\nu\{\sqrt{[-g]}\ T_\mu{}^\nu\} = 0 \qquad (24)$$

In the Yilmaz theory, $T_\mu{}^\nu$ is replaced by $\tau_\mu{}^\nu$, which is equivalent to $4\pi T_\mu{}^\nu$, and the requirement of Eq. 24 becomes

$$\partial_\nu\{\sqrt{[-g]}\tau_\mu{}^\nu\} = 0 \qquad (25)$$

Equations 24, 25 should be summed over the four values of the repeated ν index. Equation 2-35 of *5,F Addendum Appendix F* shows that Eq. 25 is always satisfied in the Yilmaz theory because of the Freud identity. Therefore, the Yilmaz theory always achieves conservation of energy and momentum of matter, including the effect of electromagnetic fields.

5, Simplification of Formula for Gravitational Potential Tensor

The discussion that followed Eq. 1 shows that this equation can be expressed as follows

$$\phi_\mu{}^\nu = f\ g_{\mu\alpha}\ (dx^\alpha/ds)\ (dx^\nu/ds)\ (dm/r) \qquad (26)$$

This formula assumes summation over the four values of the repeated α index. The "retarded" condition is still required, but the notation is dropped for simplicity. This expression should be summed over the four values of the α index. Let us indicate the summation over the index α, and replace the integral over the mass elements with a summation. This gives

$$\phi_\mu{}^\nu = \Sigma_{\Delta m}\ (\ \Sigma_\alpha\{g_{\mu\alpha}(dx^\alpha/ds)\ \}\ (dx^\nu/ds)\ (\Delta m/r)\ ) \qquad (27)$$

Equation 27 can be expressed in a more convenient form by factoring the expression ($d\tau/ds$) from each derivative, to obtain

$$\phi_\mu{}^\nu = \Sigma_{\Delta m}\ [\Sigma_\alpha\{g_{\mu\alpha}\ (dx^\alpha/d\tau)\}(dx^\nu/d\tau)\ (d\tau/ds)^2\ (\Delta m/r)\ ] \qquad (28)$$

The derivatives are now expressed directly in terms of normalized time τ. Let us implement the second summation by setting α equal to 0, 1, 2, 3:

$$\phi_\mu{}^\nu = \Sigma_{\Delta m}\{g_{\mu 0}(dx^0/d\tau) + g_{\mu 1}(dx^1/d\tau) + g_{\mu 2}(dx^2/d\tau) + g_{\mu 3}(dx^3/d\tau)\}(dx^\nu/d\tau)(d\tau/ds)^2(\Delta m/r)$$

$$= \Sigma_{\Delta m}\{g_{\mu 0} + g_{\mu 1}(V_x/c) + g_{\mu 2}(V_y/c) + g_{\mu 3}(V_z/c)\}(dx^\nu/d\tau)(d\tau/ds)^2(\Delta m/r) \qquad (29)$$

Since $x^0 = \tau$, the derivative $dx^0/d\tau$ is unity. Since $\tau = ct$, $dx^1/d\tau$ is equal to $(1/c)(dx/dt)$ which represents V_x/c, where V_x is the velocity in the x (or x^1) direction. Similarly V_y, V_z are velocities in the y (or x^2) direction, and in the z (or x^3) direction. Let us separate Eq. 29 into the two cases for $\nu = 0$, and for $\nu = k$, where k = 1, 2, 3. This gives

$$\phi_\mu{}^0 = \Sigma_{\Delta m}\{g_{\mu 0} + g_{\mu 1}(V_x/c) + g_{\mu 2}(V_y/c) + g_{\mu 3}(V_z/c)\}(d\tau/ds)^2(\Delta m/r) \qquad (30)$$

$$\phi_\mu{}^k = \Sigma_{\Delta m}\{g_{\mu 0} + g_{\mu 1}(V_x/c) + g_{\mu 2}(V_y/c) + g_{\mu 3}(V_z/c)\}(\underline{V}_k/c)(d\tau/ds)^2(\Delta m/r) \qquad (31)$$

The velocity $\underline{V}_k$ represents $\underline{V}_x$, $\underline{V}_y$, $\underline{V}_z$, for k = 1, 2, 3. This gives a convenient form for implementing the calculations on a computer.

Approximation of $\phi_\mu{}^\nu$. Comparing Eqs. 30, 31 shows that the equation for $\phi_\mu{}^k$ differs from that for $\phi_\mu{}^0$ by the factor $(\underline{V}_k/c)$. Hence if the velocity $\underline{V}_k$ of matter is much less than the speed of light c, $\phi_\mu{}^k$ is much smaller than $\phi_\mu{}^0$. Thus:

$$|\phi_\mu{}^0| >> |\phi_\mu{}^k| , \text{ for } |\underline{V}_k| << c \qquad (32)$$

When the velocity components of Eq. 30 are much smaller than the speed of light c, the equation is approximated by

$$\phi_\mu{}^0 \approx g_{\mu 0}\, \Sigma_{\Delta m}\, (d\tau/ds)^2\, (\Delta m/r) \qquad (33)$$

Hence the $\phi_\mu{}^0$ elements are approximately related by

$$\phi_k{}^0 \approx (g_{k0}/g_{00})\, \phi_0{}^0 \qquad (34)$$

In the static solution, the non-diagonal elements of the metric tensor are zero. Hence it is reasonable to expect that the non-diagonal elements of the metric tensor should be much smaller then the diagonal elements when the velocities of matter are much less than the speed of light. Hence, by Eqs. 33, 34, $\phi_0{}^0$ should be much larger than all of the other elements of the gravitational potential tensor. Consequently $\phi_0{}^0$ should be approximately equal to the trace ϕ of the tensor.

This reasoning helps to justify our use of the static Yilmaz theory solution when the velocities of matter are much smaller than the speed of light. However, to obtain a precise description of the accuracy of this approximation one should apply the equations of the time-varying Yilmaz theory in a computer solution, using the approach to be described in Section 6.

6, Computer Solution of General Time-Varying Yilmaz Theory

The general time-varying Yilmaz theory can be readily implemented on a computer. One would start with an approximate solution calculated from the static Yilmaz solution. To this one can apply Eqs. 2, 27, which are repeated as follows:

$$dg_{\mu\nu} = 2g_{\mu\nu}\, d\phi - 2\, \Sigma_{\alpha}\{\, g_{\mu\alpha}\, d\phi_{\nu}{}^{\alpha} + g_{\alpha\nu}\, d\phi_{\mu}{}^{\alpha} \} \qquad (35)$$

$$\phi_{\mu}{}^{\nu} = \Sigma_{\Delta m}\, (\, \Sigma_{\alpha}\, \{\, g_{\mu\alpha}\, (d\underline{x}^{\alpha}/ds)\, \}\, (d\underline{x}^{\nu}/ds)\, (\Delta m/r)\,)_{\text{retarded}} \qquad (36)$$

The accelerations are obtained from the geodesic equations. The general formula for the geodesic equations is given as follows by Tolman [8] (p. 495, Eq. 20):

$$d^2x^{\alpha}/ds^2 = -\, \Sigma_{\mu}\, \Sigma_{\nu}\, \Gamma_{\mu\nu}{}^{\alpha}\, (dx^{\mu}/ds\,)\, (dx^{\nu}/ds\,) \qquad (37)$$

The formula for the Christoffel symbols was given in Eq. 7 and is repeated as follows:

$$\Gamma_{\mu\nu}{}^{\alpha} = ½\, \Sigma_{\beta}\, g^{\alpha\beta}\{\partial_{\nu}g_{\mu\beta} + \partial_{\mu}g_{\nu\beta} - \partial_{\beta}g_{\mu\nu}\, \}\ . \qquad (38)$$

The contravariant metric tensor is computed from the covariant metric tensor using the following formula that was given in Eq. 17:

$$g^{\mu\nu} = -\, (-1)^{\mu+\nu}\, e^{-4\phi}\, \text{minor}[g_{\mu\nu}] \qquad (39)$$

These equations would be implemented in an iterative computer program in the following manner. The static solution is used to obtain initial approximate values for the metric tensor elements $g_{\mu\nu}$, $g^{\mu\nu}$ and the gravitational potential ϕ. Initial velocities of the bodies must be specified. The velocity and position values, along with the approximate metric tensor values are applied to Eq. 36 to obtain the elements of the gravitational potential tensor $\phi_{\mu}{}^{\nu}$. These values are applied to Eq. 35 to obtain the changes of the elements $dg_{\mu\nu}$ of the covariant metric tensor. Remember that ϕ is the sum of the diagonal elements of $\phi_{\mu}{}^{\nu}$. These changes $dg_{\mu\nu}$ are added to the approximate values of $dg_{\mu\nu}$ to obtain corrected values of the metric tenor elements.

Accelerations are computed from the geodesic equations given by Eq. 38. These are added to the velocities of the bodies to obtain new velocities. The velocities are added to the old position and time information to obtain new position and time values. Appropriate corrections are added to compensate for sampling errors.

Thus, by using conventional iterative computer techniques one can readily implement the equations for the general time-varying Yilmaz theory on a computer. This computer solution is much more complicated than the equations for the simple static Yilmaz solution. Nevertheless it is very much easier to implement than a general computer solution of the Einstein theory.

7, The Static Solution of the Yilmaz Theory

As shown by Yilmaz [6] (p. 492, three exact solutions have been derived from the general time-varying Yilmaz theory. One of these is the static solution, and the other two (which are discussed in Section 8) result in gravitational waves.

The static solution is obtained by setting to zero the derivatives relative to time. The gravitational potential tensor $\phi_\mu{}^\nu$ reduces to the single element $\phi_0{}^0$, which is equal to the trace ϕ of the tensor (the sum of the diagonal elements of the tensor). The gravitational potential ϕ at the point $\mathbf{x_p}$ is computed from

$$\phi(\mathbf{x_p}) = \Sigma\, \Delta m_k / |\mathbf{x_p} - \mathbf{x_k}| \tag{40}$$

The vector $\mathbf{x_k}$ denotes the location of the mass element Δm_k, and $|\mathbf{x_p} - \mathbf{x_k}|$ is the absolute value of the distance from the mass element to the point $\mathbf{x_p}$ where the gravitational potential is calculated. In rectangular coordinates the metric tensor is diagonal and has the following elements

$$g_{00} = e^{-2\phi} \;;\quad g_{11} = g_{22} = g_{33} = - e^{2\phi} \tag{41}$$

Let us calculate the stress-energy tensors for the static solution. In the formula for $t_\mu{}^\nu$ of Eq. 11, set $\phi_\mu{}^\nu$ equal to $\phi_0{}^0$ which is equal to ϕ. This gives

$$\begin{aligned} t_\mu{}^\nu &= - 2\partial_\mu\phi\partial^\nu\phi + \partial_\mu\phi\partial\phi + \delta_\mu{}^\nu\{\partial_\lambda\phi\partial^\lambda\phi - \tfrac{1}{2}\,\partial_\lambda\phi\partial^\lambda\phi\} \\ &= - \partial_\mu\phi\, \partial^\nu\phi + \tfrac{1}{2}\, \delta_\mu{}^\nu\, \Sigma_\lambda\, \partial_\lambda\phi\, \partial^\lambda\phi \end{aligned} \tag{42}$$

For clarity, the implied summation over the index λ is indicated. Equation 12 for $\tau_\mu{}^\nu$ can be written as

$$\tau_\mu{}^\nu = \partial_\alpha\partial^\alpha\phi_\mu{}^\nu - \partial_\alpha\partial^\nu\phi_\mu{}^\alpha + 2\partial_\alpha\phi\,\partial^\alpha\phi_\mu{}^\nu - 2\partial_\alpha\phi\,\partial^\nu\phi_\mu{}^\alpha \tag{43}$$

Since the tensor $\phi_\mu{}^\nu$ reduces to the single element $\phi_0{}^0$, the index α in the second and fourth terms must be zero. Since these terms involve differentiation relative to time (index 0), the second and fourth terms are zero. Since the first and third terms contain the tensor $\phi_\mu{}^\nu$, the indices μ and ν must be zero. Therefore the tensor $\tau_\mu{}^\nu$ given by Eq. 43 reduces to a single element $\tau_0{}^0$, which is equal to

$$\tau_0{}^0 = \partial_\alpha\partial^\alpha\phi_0{}^0 + 2\,\partial_\alpha\phi\,\partial^\alpha\phi_0{}^0 = \Sigma_\alpha\,(\partial_\alpha\partial^\alpha\phi + 2\,\partial_\alpha\phi\partial^\alpha\phi) \tag{44}$$

The implied summation over the index α is indicated. This is the only nonzero element of the stress-energy tensor for matter $\tau_\mu{}^\nu$ for the static Yilmaz solution. It can be shown that this formula for $\tau_0{}^0$ reduces to

$$\tau_0{}^0 = - e^{-2\phi}\, \nabla^2\phi \tag{45}$$

The Laplacian $\nabla^2\phi$ is defined as follows in *rectangular coordinates:*

$$\nabla^2\phi = (\partial^2\phi/\partial x^2) + (\partial^2\phi/\partial y^2) + (\partial^2\phi/\partial z^2) \tag{46}$$

8, Gravitational Wave Solutions of the Yilmaz Theory

Besides the static solution, the general Yilmaz theory yields two other specific solutions, which represent gravitational waves. In one of these gravitational wave solutions, the gravitational potential tensor $\phi_\mu{}^\nu$ has two nonzero components, which are related by

$$\phi_1{}^1 = -\phi_2{}^2 = \zeta(\tau, z) \tag{47}$$

The nonzero elements of the metric tensor are

$$g_{11} = -e^{4\zeta};\ g_{22} = -e^{4\zeta} \tag{48}$$

As shown in Eq. 47, the variable ζ varies with time τ and with position z in the z-direction. This variable can be expressed as

$$\zeta(\tau, z) = \Sigma_k (Ae^{ikx} + A^*e^{-ikx}) \tag{49}$$

This is a general expression for a wave that is propagating in the x-direction. This is expressed in terms of the complex variable i. This represents a transverse gravitational wave that is propagating in the x-direction.

In the second gravitational wave solution, the gravitational potential tensor $\phi_\mu{}^\nu$ has two nonzero components, which are related by

$$\phi_1{}^2 = -\phi_2{}^1 = \xi(\tau, z) \tag{50}$$

The nonzero elements of the metric tensor are

$$g_{11} = g_{22} = -\cosh[4\xi];\quad g_{12} = g_{21} = -\sinh[4\xi] \tag{51}$$

As shown in Eq. 50, the variable ξ varies with time τ and with position z in the z-direction. This variable can be expressed as

$$\zeta(\tau, z) = \Sigma_k(Be^{ikx} + B^*e^{-ikx}) \tag{52}$$

This also is a transverse gravitational wave that is propagating in the x-direction.

9, Conservation of Energy and Momentum in Einstein and Yilmaz Theories

Let us review the gravitational field equations of the Einstein and Yilmaz theories, which are as follows:

Einstein theory: $G_\mu^{\ \nu} = R_\mu^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu} R = -8\pi T_\mu^{\ \nu} = -2\tau_\mu^{\ \nu}$ (53)

Yilmaz theory: $G_\mu^{\ \nu} = R_\mu^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu} R = -2(\tau_\mu^{\ \nu} + t_\mu^{\ \nu})$ (54)

The Bianchi identity shows that the covariant derivative of $R_\mu^{\ \nu}$ is equal to the covariant derivative of $\frac{1}{2}\delta_\mu^{\ \nu} R$, and so the covariant derivative of the difference $(R_\mu^{\ \nu} - \frac{1}{2}\delta_\mu^{\ \nu} R)$ is zero. This means that the covariant derivative of the Einstein tensor $G_\mu^{\ \nu}$ is zero, as shown by

$$D_\nu G_\mu^{\ \nu} = 0 \tag{55}$$

The symbol D_ν denotes covariant derivative. The covariant derivative is explained in Appendix J of *Believe* [1]. Applying Eq. 55 to Eqs. 53, 54 shows that

Einstein theory: $D_\nu T_\mu^{\ \nu} = D_\nu \tau_\mu^{\ \nu} = 0$ (56)

Yilmaz theory: $D_\nu(\tau_\mu^{\ \nu} + t_\mu^{\ \nu}) = 0$ (57)

As shown in Section 5.4, the Freud identity insures that the following condition is satisfied by the Yilmaz theory:

$$\partial_\nu\{\surd[-g]\,\tau_\mu^{\ \nu}\} = 0 \tag{58}$$

This implies summation over the ν index. Landau and Lifshitz [7] (p. 280) explain that conservation of energy and momentum for matter (including electromagnetic energy) requires that the following integral must be conserved: $\surd f[-g]\tau_\mu^{\ \nu} dS_\nu$, where $\surd[-g]\,\tau_\mu^{\ \nu}$ is the tensor density of the energy-momentum tensor $\tau_\mu^{\ \nu}$. They state that to achieve this requires that Eq. 58 be satisfied. Since this equation is always satisfied in the Yilmaz theory, conservation of energy and momentum for matter (including electromagnetic energy) is guaranteed in the Yilmaz theory.

In terms of the symbolism of the Einstein theory this requirement becomes

$$\partial_\nu\{\surd[-g]\,T_\mu^{\ \nu}\} = 0 \tag{59}$$

This condition is not automatically satisfied in the Einstein theory. However the energy-momentum tensor $T_\mu^{\ \nu}$ must be designed to achieve conservation of energy and momentum of matter when the Einstein theory is appropriately applied. To achieve this requires that Eq. 59 must be satisfied.

Let us compare Eqs 56 and 59. Because of the Bianchi identity, the gravitational field equation of the Einstein theory requires that the covariant derivative of the energy-momentum tensor $T_\mu{}^\nu$ must be zero. However, to achieve conservation of energy and momentum of matter, the energy-momentum tensor must be designed so that the partial derivative of $\sqrt{[-g]}T_\mu{}^\nu$ is zero. This indicates that conflicting requirements are placed on the energy-momentum tensor $T_\mu{}^\nu$, and so the equations of the Einstein theory are *over-constrained*. Because of this property, the Einstein theory can yield conflicting solutions.

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