5,E Addendum Appendix E
Apparent Distance in Yilmaz Cosmology Model

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In Appendix C of *Believe* [1], Table C-2 gives the following for the apparent increment of radial distance Δr_{ap} observed on earth (at r = 0) that corresponds to an increment of true radial distance Δr measured at the true distance r.

$$\Delta r_{ap} = \Delta r \exp[-(r/r_o)^2/2] \qquad (1)$$

Replace the incremental distances (Δr_{ap}, Δr) by the derivatives (dr_{ap}, dr) and integrate. This gives the apparent radial distance r_{ap} measured by the earth observer, which is

$$r_{ap} = \int d\, r_{ap} = \int \exp[-(r/r_o)^2/2]\, dr \qquad (2)$$

Divide this by r_o, and place limits on the integral, to obtain

$$r_{ap}/r_o = \int_{(0,z)} \exp[-(r/r_o)^2/2]\,(dr/r_o) = \int_{(0,z)} \exp[-x^2/2]\, dx \qquad (3)$$

The subscript for the integral sign indicates that the lower limit on the integral is zero, and the upper limit is z. The value for the upper limit z is

$$z = (r/r_o) \qquad (4)$$

The integral of Eq. 3 is related to the *normal distribution function* denoted $\Phi(x)$, which is used extensively in probability and statistics. As shown by Feller [5] (p. 164), the normal distribution function is defined as

$$\Phi(x) = (1/\sqrt{[2\pi]}) \int_{(-\infty, x)} \exp[-y^2/2]\, dy \qquad (5)$$

The lower limit on this integral is minus infinity ($-\infty$) and the upper limit is (x). Values of this function $\Phi(x)$ are tabulated on page 167 of Ref. [4]. The derivative of the *normal distribution function* $\Phi(x)$ is denoted $\phi(x)$, and is called the *normal density function*. The density function $\phi(x)$ is denoted by the lower case Greek phi, whereas the distribution function $\Phi(x)$ is denoted by the upper case Greek phi. The *normal density function* $\phi(x)$ is defined as

$$\phi(x) = d\Phi(x)/dx = (1/\sqrt{[2\pi]}) \exp[-x^2/2] \qquad (6)$$

This is the common bell-shaped curve used in statistics. Since the area under this curve is unity, and the curve is symmetric about zero, the value of its integral $\Phi(x)$ must be unity at the value ($x = \infty$), and must be 0.5 at x = 0. Hence, Eq. 5 can be expressed as follows in terms of an integral from zero to x:

$$\Phi(x) = 0.5 + (1/\sqrt{[2\pi]}) \int_{(0,\, x)} \exp[-y^2/2]\, dy \qquad (7)$$

Multiply Eq. 7 by 2, and solve for the integral. This gives

$$\int_{(0,\, x)} \exp[-y^2/2]dy = \sqrt{[\pi/2]}\{ 2\Phi(x) - 1\} = \sqrt{[2\pi]}\{ \Phi(x) - 0.5\} \qquad (8)$$

Applying Eq. 8 to Eq. 3 allows us to express the apparent radial distance r_{ap} as follows, in terms of the normal distribution function Φ:

$$r_{ap}/r_o = \int_{(0,\, z)} \exp[-x^2/2]\, dx = \sqrt{2\pi}\{\Phi[r/r_o] - 0.5\} \qquad (9)$$

The upper limit of the integration (z) is equal to (r/r_o). The values of $\Phi(x)$ are obtained from tabulated data given by Feller [5] (p.167). From these one can compute from Eq. 9 the values of the function r_{ap}/r_o listed in Table 1.

Table 1: *Values of the apparent distance ratio r_{ap}/r_o of a galaxy versus the true distance ratio r/r_o derived from Eq. 9, using tabulated values of the normal distribution function $\Phi(x)$.*

x, r/r_o	$\Phi[x]$	r_{ap}/r_o	x, r/r_o	$\Phi[x]$	r_{ap}/r_o
0	.500 000	0	2.0	.977 250	1.196 288
.1	.539 828	.099 833	2.1	.982 136	1.208 535
.2	.579 260	.198 675	2.2	.986 097	1.218 464
.3	.617 911	.295 559	2.3	.989 276	1.226 433
.4	.655 422	.389 585	2.4	.991 802	1.232 765
.5	.691 462	.479 924	2.5	.993 790	1.237 748
.6	.725 747	.565 864	2.6	.995 339	1.241 631
.7	.758 036	.646 800	2.7	.996 533	1.244 624
.8	.788 145	.722 272	2.8	.997 445	1.246 910
.9	.815 940	.791 944	2.9	.998 134	1.248 637
1.0	.841 345	.855 625	3.0	.998 650	1.249 930
1.1	.864 334	.913 250	3.1	.999 032	1.250 88
1.2	.884 930	.964 876	3.2	.999 313	1.251 592
1.3	.903 200	1.010 673	3.3	.999 517	1.252 103
1.4	.919 243	1.050 886	3.4	.999 663	1.252 469
1.5	.933 193	1.085 854	3.5	.999 767	1.252 730
1.6	.945 201	1.115 953	3.6	.999 841	1.252 916
1.7	.955 439	1.141 616	3.7	.999 892	1.253 043
1.8	.964 070	1.163 251	3.8	.999 928	1.253 134
1.9	.971 283	1.181 331	∞	1.00000	1.253 314

References

[1] Adrian Bjornson, *A Universe that We Can Believe*, *Addison :Press, 2000, described in world-wide website Olduniverse.com* ISBN 09703231-0-7.

[5] William Feller, *An Introduction to Probability Theory and its Applications*, *vol. 1*, 2nd Ed., John Wiley, New York, 1959, Library of Congress, 57-10805.